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Economics 105
September 4, 2000r

Problem Set #1: Production Possibilities

Due: Friday, September 8th! You must submit answers to questions 1 and 2 by this date; if necessary, you may submit the remainder of the problem set as late as 9:00 AM on Monday, September 11th.

Please turn in your problem set at the beginning of class on the date due or deposit it in the Economics 105 slot in the alcove around the corner to the left of the Economics Department Office, Room PAC 123 by 3:00 P.M.

Read: Before starting this problem set, be sure to read Lovell Ch 2: Production Possibilities (Note: this reading is in the Course Packet; or it may be downloaded from the Econ 105 Home Page.

Check List:

(a) Need help? See the instructor or the TA.
(b) Be neat! (Full name at top of paper. Ball point pen or dark pencil.)
(c) Show calculations so that you will not lose too many points for numerical errors.

Grading: Almost all problem sets will earn a ★ (star grade), signifying eminently satisfactory work. An exceptional few may earn a ★+ (star plus) indicating a truly outstanding paper, including strong answers to any optional or Honors Option Questions. Let us hope that no one will receive a ★– (star minus) warning of serious problems, an **S** for Sloppy work, a **T** for tardy, or an **M** for missing. **T** and **M** papers will be penalized, particularly if there is more than one during the course of the semester.

1. In his study of the supply of jet fuel for the Korean War, Allan Mann suggested that enemy action might shift the production transformation curve as illustrated on the table.

Reduced Refining Capacity

	JP-4 jet fuel (Millions of barrels per day)	Petroleum-non jet fuel (Millions of barrels per day)
a	0.00	3.28
b	0.50	3.17
c	1.00	2.58
d	1.50	1.65
e	1.64	1.00
f	1.71	0.38

a. Plot the reduced capacity data points on a sheet of graph paper, forming a production possibilities curve with non-jet (civilian) fuel on the abscissa and jet fuel on the ordinate. [Suggestions: Label the axes of your graph carefully; mark each data point with the appropriate letter — a, b, c, etc.; connect adjacent points with straight lines.]

b. If 3.00 million barrels of Petroleum non-jet fuel are being produced, how much JP-4 Jet fuel can be refined? What is the MRT involved in producing an additional barrel of jet fuel when the output of non-jet fuel is 3.00 million barrels per day? What is the opportunity cost of an additional barrel of jet fuel?

c. What is the opportunity cost of an additional barrel of civilian output (non-jet fuel) in terms of foregone barrels of jet fuel when civilian output is 3.00 million barrels? Explain how the

opportunity cost of civilian fuel is related to the opportunity cost of jet fuel and the slope of the production transformation curve.

2. Suppose that the production possibilities for goods x and y are specified by $x^2+y^2 = 16$, $x > 0$, $y \geq 0$.
   a. Plot the production transformation curve.
   b. Find the equation for y as a function of x. Then find dy/dx.
   c. Determine the marginal rate of transformation and the opportunity cost of x (in terms of foregone y output) when x = 2.
   d. Determine the quantity of x and y that will maximize total revenue if $p_x = p_y = \$1.00$
   e. Derive the **supply curve for x**. That is to say, find the quantity of x produced as a function of p_x/p_y.

   [Note: You should treat A as a continuous variable, like milk, so you can solve this problem with the calculus; do not restrict your answers to integer values.]

3. Congratulations, you have inherited a boat and $1,000 from your late Uncle Jake. In the United States corn sells for $1.00 per bushel and wheat for $2.00 a bushel. In England, corn and wheat both sell for one British pound a bushel.
   a. Explain how you can double your $1,000 through an artful arbitrage operation, assuming that shipping costs are negligible. [Hint: Look up "arbitrage" in the *EconAE* glossary.]
   b. If a large number of traders attempt to profit in this way, will the opportunity for profit be diminished? Why.

   [Note: you do *not* have to know the foreign exchange rate in order to answer this question].

5. *Optional but fun:* In his 1817 treatise, David Ricardo presented the following numerical example, which is similar to what is now known as a "linear programming problem."

   > Suppose that 80 English workers can produce 80 barrels of wine or 100 bolts of cloth in one year but that in Portugal 60 workers can produce 120 barrels of wine or 60 bolts of cloth. In England they are producing 40 barrels of wine and 50 bolts of cloth while in Portugal they produce 60 barrels of wine and 30 bolts of cloth.

   Assuming, as did Ricardo, that the relationship is linear, the production transformation curve with 80 English workers is the function $W_E = T_E(C_E) = 80 - 0.8C_E$.
   a. Plot the production transformation curve for England.
   b. Determine the production transformation curve for Portugal's 60 workers; plot it on a separate graph.
   c. If both countries concentrate on producing only wool, what is the maximum quantity they can produce? If they concentrate in both countries on producing only wine, how much can they produce (assuming workers stay sober)?
   d. Determine a price ratio for wine and wool in England (p_w^E/p_c^E) that would induce maximizing British farmers to produce both wine and cloth. Is England producing efficiently (i.e., is it operating on its production possibility frontier)? What price ratio (p_w^P/p_c^P) must prevail in Portugal if maximizing producers are to decide to produce both wine and cloth? Is Portugal operating efficiently?
   e. Suppose you have 10 bolts of cloth in your London warehouse. The price ratios you determined in c prevail in both England and Portugal. How much wine can your cloth be exchanged for in England? Would you do better to ship your cloth to Portugal, buy wine there, and bring it home to sell it in England. How much profit would you make from the round trip?
   f. Determine the production possibility curve for the combined output of England and Portugal; plot it on a graph.

   Hints: This is not a calculus problem.
   More Hints for question are on the next page:

Since the production transformation curve for England is $W_e = 80 - .8C_e$, the opportunity cost of a bolt of cloth in England is .8 barrels of wine. (Therefore, the opportunity cost of a barrel of wine is 1/0.8 = 1.25 bolts of cloth).

If revenue maximizing producers in England are producing both commodities, a bottle of wine must cost 1.25 bolts of cloth. If wine were cheaper, it would pay to produce only cloth. If wine were more expensive, it would pay to produce only wine. This may be visualized on a graph of England's production transformation curve by adding appropriate iso-revenue lines.

The production transformation curve for Portugal is $W_p = 120 – 2C_p$; therefore, the opportunity cost of one bolt of cloth is two barrels of wine. Thus the opportunity cost of cloth is 2 barrels of wine in Portugal.

Ricardo would say that England has a "comparative advantage" in the production of cloth because the opportunity cost of producing cloth in England of 0.8 barrels of wine is less than the opportunity cost of producing cloth in Portugal (2 barrels of wine in Portugal). But because the opportunity cost of producing wine is the reciprocal of the opportunity cost of producing cloth, Portugal must have a comparative advantage in producing wine.

In thinking about question 5e you will find it helpful to consider the following four feasible allocations:

	Cloth Eng	Wine Eng	Cloth Port	Wine Port	Cloth Total	Wine Total
#1	100	0	60	0	160	0
#2	0	80	0	120	0	200
#3	100	0	0	120	100	120
#4	0	____	60	____	60	____

Production is efficient in each country (both are operating on their production possibility frontier).

If you plot Wine Total on Cloth Total you will have three points on the world production possibility frontier and one obviously inefficient point.

If you draw straight lines connecting the three efficient points you will have the world production possibility frontier.

It would be inefficient to have England producing any wine if Portugal is producing any cloth. Why? Because if England is producing some wine, she could get an extra bolt of cloth by cutting wine output by 0.8 of a barrel. Then Portugal could cut cloth output by 1 bolt and generate 2 barrels of wine. Thus the world would have just as much cloth and – 0.8 + 2 = 1.2 more barrels of wine!

Ricardo's point was that trade can benefit two countries that are producing the same two commodities unless the opportunity costs are the same in both countries. He was implicitly assuming that both countries were operating on their production possibility frontier (i.e. efficiency and full employment.

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6. Honors Option: A Two Country World

Country 1 has the following production transformation curve: $x_1 + y_1 = 5$

Country 2 has the following production transformation curve: $x_2^2 + y_2^2 = 25$

Find the production transformation curve for the combined output of the two countries.

Hint: Invoke the efficiency proposition on page 13 of Chapter 2.

7. Honors Option: Another Two Country World

Country 1 has the following production transformation curve: $x_1^2 + y_1^2 = 4$

Country 2 has the following production transformation curve: $x_2^2 + y_2^2 = 16$

Find the production transformation curve for the combined output of the two countries.

Hint: Invoke the efficiency proposition on page 13 of Chapter 2.
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