Economics 105 Mike Lovell
Due: 11:00 AM, Wednesday, October 11, 2000

Problem Set #4: Production Functions; Cost Functions

Note: Before starting this exercise you must read Chapters 5: The Business Enterprise: Theory of the Firm; study the graphs in the packet along with the Chapter in the course book.

I. Accounting:

Here is Table 5.2: Income Statement of the Fly-by-Nite Aircraft Company

January 1, 2001 to December 31, 2001
(all figures in $1,000 of dollars)

Net Sales			**$800**	
Less:				
Cost of Manufacturing				
Materials	**$200**			
Labor	**250**			
Depreciation Expense	**100**			
	$550			
Less Inventory Increase	**(100)**			
Cost of Goods Sold		**(450)**		
Gross Margin			**350**	
Less Selling Cost	**30**			
Interest Expense	**20**			
			(50)	
Profits			**300**	
Less Corporate Profit Tax			**(105)**	
Profits After Taxes			**195**	
Less Dividends			**(100)**	
Addition to Retained Earnings			**$95**	

1. On the basis of its December 31, 2000 balance sheet (Table 5.1) and the information in 5.3.1 used for year 2001 income statement, draw up the firm's balance sheet for December 31, 2001.

2. A new accountant hired by the firm recommends two changes in accounting practice. First, he argues that the firm should employ accelerated depreciation, which will increase its depreciation charge for year 2001 by $100. Second, he recommends that the firm adopt LIFO inventory.

To illustrate, he says, consider the stock of lubricating oil, which was valued at $100,000 as of on December 31, 2000. This stock consisted of 100,000 gallons purchased at a price of 50¢ per gallon on June 31, 2000, plus an additional 50,000 gallons purchased at a price of $1.00 per gallon on December 15th, 2000. During 2001 the firm used up 100,000 gallons of oil. What was the value of that oil?

FIFO ACCOUNTING: Under FIFO (First in First Out) the accountant had evaluated the entire 100,000 gallons used up at 50¢ per gallon and the remaining stock of 50,000 gallons at $1.00, entering a figure of $50,000 on the firm's balance sheet for oil at the end of the year.

LIFO ACCOUNTING: Instead, the new accountant argues, the firm should adopt LIFO (Last in First Out) accounting: The first 50,000 of oil that was used up is evaluated at the most recent purchase price of $1.00 per gallon and the remaining at 50¢ per gallon. Ass a result oil will contribute $75,000 rather than $50,000 to the firm' inventory increase but leave the remaining 50,000 gallons evaluated at 50¢ per gallon on the firm's balance sheet.

How would the firm's accounting profits change if these two procedures were adopted? Should it adopt accelerated depreciation? Explain why or why not. Should the firm switch from FIFO to LIFO inventory accounting? Why or why not?

II. Cost and Production Functions: A firm uses two inputs, physical capital K (machinery, buildings and other capital equipment) and L (hours of labor) to produce output q per hour. The Cobb-Douglas production function is

$q = K^{0.5}L^{0.5}$

1. Suppose our firm has 1 machine and employs 16 workers. What is output per hour, the average product of labor, the marginal product of labor, and the marginal product of capital? In general, find equations for each of these three concepts as a function of L and K.

2. Is this production function homogeneous of degree 1? Show why.

3. Suppose workers earn $10.00 per hour and the firm has only one machine, which costs $20 per hour to operate.
   a. How much labor will our firm have to hire to produce 10 units of output per hour? What will be the total cost to our enterprise of producing 10 units of output per hour? What is average (total) cost? What is average variable cost?
   b Now derive the firm's total cost function, C(q), showing how the total costs depend on the level of output per hour, given that the firm has only one machine. Note that this is referred to as the "short-run" total cost function because the firm does not have enough time to adjust the number of machines. Hint: First find the function showing labor as a function of q, given that K = 1.
   c Determine the (short run) marginal cost, the average cost, and the average variable cost functions, given that K = 1.

4 The long run
   a. In the *long run* the firm has sufficient time to adjust the number of machines. What is the least cost technique for producing 10 units of output, given that workers earn $10.00 per hour and machines cost $20 per hour to operate? That is to say, find the quantities of labor and capital we should employ and their total cost, if we are to produce 10 units of output at minimum cost. What will be cost per unit (average cost) when output is 10?
   b. What is the minimum average cost if the long run level of output is 20? (Shortcut: Note that the production function is homogeneous of degree 1).
   c. Determine the firm's long-run total cost function, given that it can have as many machines as it wants at a cost of $20 per machine.

5. Short Run Supply: Suppose our enterprise has but one machine (K=1). It can sell its output on a competitive market at a price of $30. Our firm wishes to maximize profits $\pi(q) = R(q) - C(q)$, where total revenue is $R(q) = pq$ and C(q) is the total cost function.
   a. Determine the profit maximizing level of output, total costs, total revenue and profit, given that K = 1.
   b. Determine marginal revenue and marginal cost at the profit maximizing level of output you found in 5a. Is it true for this example that marginal revenue is equal to marginal cost at the profit maximizing level of output?
   c. Suppose instead of selling its output in a competitive market the firm faces the demand curve $q = 50 - p$. What level of output q will maximize profits? What price should the firm charge?

6. Management Science
   a. Use the Optimal Lot Size equation (square root rule of section 5.6.1) to check whether the consultant has indeed calculated the appropriate lot size for Table 5.4.
   b. Determine how the optimal lot sizes recorded on the table would change if the inventory carrying cost for Good A were $15, unchanged for Good B, and only $8 for Good C?
   c. Observe on that table that the setup size recommended by the consultant yields annual setup costs that are precisely equal to annual inventory carrying cost. Is this a coincidence, or can you show it must always be the case when the optimal lot size is being produced?

7. On the Road: The following table about the cost of driving a car is reproduced from the Construct the best estimate you can of the cost function for driving a car, assuming it is the simple linear form

   Total Cost = $\alpha + \beta$ mileage.

   a. Find the marginal cost of driving the car an extra mile.

   b . Joe Carman offers to drive a Wesleyan classmate to the airport (30 miles each way) in his 1987 Honda for $15. Is he making money? If you drive me to the airport, should you charge me marginal cost, average cost, or the taxi fare? Explain.

Cost of owning and driving a Car, 1996
(assuming the car is driven 10,000/year[1])

Item	Unit	
Cost per mile	Cents	51.43
Variable cost	Cents/mile	10.10
Gas and Oil	Cents/mile	5.90
Maintenance	Cents/mile	2.80
Tires	Cents/mile	1.40
Fixed Cost	Dollars	4,997
Insurance	Dollars	782
License & registration	Dollars	229
Depreciation	Dollars	3,208
Finance Charges	Dollars	778

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Honors Option: A CES (Constant Elasticity of Substitution) production function has the form

$$q(K,L) = [\beta K^{\rho} + (1-\beta)L^{\rho}]^{\varepsilon/\rho}.$$

Here β is the distribution parameter determining the relative importance of capital and labor, ε is the elasticity of substitution, and ρ is called the scale parameter.[2]

a. What is the marginal product of labor?

b. For what values of β, ε, and ρ does this function exhibit constant returns to scale?

[1] *Statistical Abstract of the United States*, 1998, Table 1025.

[2] This function was introduced into the literature by K. J. Arrow, H. B. Chenery, B. S. Minhas and R. M. Solow, "Capital-Labor Substitution and Economic Efficiency," *Review of Economics and Statistics (1961).*
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