Problem Set #3: Utility

Due: 11:00 AM, Friday, September 29th. Please read Ch4: Maximizing Satisfaction in volume 2 of the Course Book in conjunction with this problem set.

Note: Part A: If you turn in Part A = Questions 1 to 3, by 11:00 AM on Friday, September 22nd, we will return your answers to you before the Quiz on Wednesday, September 27th. Otherwise, turn it in on the 29th.

1. There are 100 units of the commodity, call it X, to be divided between two individuals, Albert and Baker.

Suppose that Albert's utility function is $U_a(X_a) = X_a^{0.5}$

and Baker's is $U_b(X_b) = 5 + X_b^{0.5}$.

Note: As usual, assume that the commodity X is a liquid, so that X_a and X_b can be any nonnegative real number.

a. If Albert enjoys 5 utiles of satisfaction, how much must he be consuming? How much is left for Baker to consume? How much utility can Baker enjoy if Albert is enjoying 5 utiles of satisfaction?
b. Plot on a graph the "utility possibility frontier" showing (on the ordinate) how much utility Baker can enjoy as a function of the utility enjoyed by Albert (plotted on the abscissa).
c. Following Jeremy Bentham, determine the allocation of good X that will maximize the "sum total of happiness" or $S = U_a(X_a) + U_b(X_b)$. How much utility will each of our citizens enjoy as a result of this allocation?
d. Following John Rawls, find the allocation of resources that will "maximize the position of the least advantaged;" i.e., find $\max\{\min[U_a(X_a),U_b(X_b)]\}$ How much utility will each of the individuals enjoy as a result of this allocation? (Approximate answers read off the graph will suffice.)

2. Consider Mary, a utility maximizer consuming goods X_1 and X_2 at prices p_1 and p_2. Her income is M dollars. Suppose her utility function is

$$U = 2X_1^{0.5} + X_2$$

Solve for the demand function for X_1 showing how the quantity consumed of this commodity depends on the price of the two goods and income. Note that you are to maximize U subject to both the income constraint $M = p_1X_1 + p_2X_2$ and the sign constraints $X_1 \geq 0$ and $X_2 \geq 0$.

Caution: Why does it make a difference if $(p_2/p_1)^2 \geq M/p_1$?

3. Mary Continued:
   a. Derive the *indirect utility function* showing how Mary's utility depends upon her money income and the prices of the two commodities; i.e., find a function of the form $U = v(M,p_1,p_2)$.

   b. Find from the indirect utility function the *expenditure function* showing how much income Mary must have to achieve a specified level of utility, given prices; i.e., find a function $M = E(U, p_1,p_2)$

Part B: Due by 11:00 AM, Friday, September 29th.

4. Suppose that Alfred has the utility function $U_a(X,Y) = 3\log_e X + \log_e Y$ while Barbara has utility function $U(X,Y) = X^3Y$.
   a. Does Alfred's utility function satisfy the Law of Diminishing Marginal Utility? Does Barbara's? Explain why or why not.

b. Forget about Diminishing Marginal Utility for the moment. Just apply the calculus to find Alfred's demand function for X as a function of income, the price of X, and the price of Y. Now find the demand function for Y. Are these demand functions reasonable?

c. Now find Barbara's demand function for X. Compare her demand function with Alfred's.

d. Suppose that empirical research showed that Charles, who consumes only commodities X and Y, has demand function $X = ¾M/p_x$ and $Y = ¼\ M/p_y$. Determine if you can whether his utility function is of the form $U(X,Y) = X^3Y$, $U(X,Y) = 3\log_e X + \log_e Y$, or $U(X,Y) = X^{3/4}Y^{1/4}$ and, if possible, whether his utility function obeys the law of diminishing marginal utility. Explain. If you cannot answer these questions on the basis of the empirical evidence about Charles demand function, is there any way in which it could be determined empirically whether his utility is subject to the law of diminishing marginal utility? Discuss

5. Diana's demand function for widgits is of the form $W = 50 + M/p_w - 2p_x$ and her brother Edward has demand function $W = 4M/p_w + (p_x/p_w)^{0.5}$

a. Is Diana's demand function homogeneous of degree zero in income and prices? Is Edwards?

b. Explain why Diana cannot be a utility maximizer.

6. A worker's income is M = wh, where w is the hourly wage and h is the number of hours worked; the worker enjoys L = 24 – h hours of leisure each day. Our worker's utility function is $U = M + 16L^{1/2}$, where M is daily income spent on consumption goods and L is hours of leisure per day.

a. Determine the labor supply function showing the number of hours our utility maximizer will work as a function of the wage rate, where obviously, $0 \leq L \leq 24$.

b. Find out if and how the number of hours worked will change if the utility maximizer receives an inheritance yielding an income of $10 per day.

c. How will the supply of labor function change if he receives a welfare benefit B = max($10-M,0).

Note: This is an example of a "constant marginal utility of income" utility function (the marginal utility of income, $\partial U/\partial M$, is a constant).

7. Draw indifference maps illustrating each of the following possible outcomes:

a An increase in the price of a commodity leads to an *increase* in the quantity consumed, given money income and all other prices.

b. An increase in the tax on wage income leads to an increase in hours worked.

c. An increase in interest rates leads to a reduction in saving.

8. In England during World War II, the slogan "fair shares for all" was popularized in order to encourage patriotic citizens to abide by the strict rationing of basic foodstuffs that was imposed in an effort to cope with severe wartime shortages. Thus if there are 10 units of X and 20 of Y to be allocated between Dick and Jane we would have $X_D = X_D = 5$ and $Y_D = Y_J = 10$. Show on a graph that while the equal division might be considered fair, it does not necessarily yield an efficient allocation of resources. Hint you are to construct a box diagram with an appropriate banana shaped region.

Honors Option: The Stone-Geary utility function is of the form

$$U(X_1, X_2) = (X_1 - b_1)^{a_1}(X_2 - b_2)^{a_2}$$

where $\beta_1 \geq 0$ and $\beta_2 \geq 0$ are interpreted as subsistence or "wolf-point" levels of consumption.

a. For what values of the parameters α_1, α_2, β_1, and β_2 does the Stone-Geary utility function reduce to equation 3.1 of Ch 4: Maximizing Satisfaction?

b. Derive the demand function for X_1 implied by the Stone-Geary utility function. [Hint: simplify the problem by working with the transformed variables $X_1^* = X_1 - \beta_1$, and $X_2^* = X_2 - \beta_2$.]

c. Prove that the demand for X_1 is homogeneous of degree 0 in income and prices.

Note: The Stone-Geary utility function yields a demand function that is linear in prices and incomes, which facilitates its estimation from empirical data by econometric techniques.