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ECON 110-02

OK!!!

# Chapter 12 Problem Set

1. $g = 4\%$, $k = 5\%$, $n = 3\%$

$\ln(Q/K) = \ln Q - \ln K$

$\frac{d(\ln(Q/K))}{dt} = \frac{d(Q/K)/dt}{Q/K} = \frac{dQ/dt}{Q} - \frac{dK/dt}{K} = g - k = 4\% - 5\% \boxed{= -1\%}$

rate of per capita output growth $= g - n = 4\% - 3\% \boxed{= 1\%}$ ?

See fn 3, page 561

2. $Q = (1.02)^t L^{2/3} K^{1/3}$

$s = 10\%$, $n = 1\%$

$\frac{8\frac{1}{3}\%}{2/3} = 4.9$

a) $g^e = \frac{\rho + \lambda n}{1 - \gamma} = \frac{2\% + (2/3)(1\%)}{1 - 1/3} = \frac{2\frac{2}{3}}{2/3} = \frac{1}{2}(8.01) \boxed{= 4.005\%}$

b) $(Q/K)^e = g^e/s = \frac{4.005}{10\%} \boxed{= 0.04005}$ ✓

c) equilibrium growth rate of per capita income $= g^e - n = 4.005\% - 1\% \boxed{= 3.005\%}$ ✓

3. $i = 10\%$, $P_{oil} = \$20$, 500,000 barrels

✓ a) If the price of oil next year is expected to increase at a rate greater than the interest rate, I would pump none of the oil. (By waiting until next year – or later – I would be able to make more money by pumping the oil than at the present time. If I invested the money now, I would make \$10,000,000 (1.1) = \$11,000,000, but if I waited a year, I would make \$22.40(500,000) = \$11,200,000.) Since my profits will be higher if I wait a year, I would pump none of the oil this year.

✓ b) If oil well owners can make higher profits by pumping the oil now and investing it, they will pump all of the oil; if they can make higher profits by waiting a year, they will pump none of the oil. If they pump only some of the oil, they must expect that they would make equal profits by pumping the oil now and investing the money or by waiting. (For example, suppose $p = \$20$ and $i = 10\%$. If $p$ is expected to increase 10% next year, they could make \$10,000,000(1.1) = \$11,000,000 by pumping this year and \$22(500,000) = \$11,000,000 by waiting.) Basically, the oil well owners are assuming that the price of oil will increase at a rate equal to the interest rate – in the above example, 10%.

4. $R(F_{t-1}) = (25-(F_{t-1}-5)^2)^{1/2} - 1$

a) $R(0) = (25-(0-5)^2)^{1/2} - 1 = \sqrt{25-25} - 1 = 0 - 1 \boxed{= -1}$

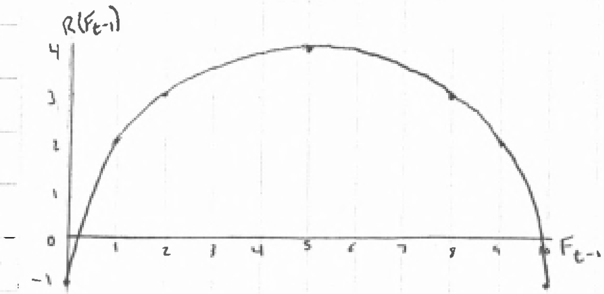
$R(5) = (25-(5-5)^2)^{1/2} - 1 = \sqrt{25-0} - 1 = 5 - 1 \boxed{= 4}$

$R(8) = (25-(8-5)^2)^{1/2} - 1 = \sqrt{25-9} - 1 = 4 - 1 \boxed{= 3}$

$R(9) = (25-(9-5)^2)^{1/2} - 1 = \sqrt{25-16} - 1 = 3 - 1 \boxed{= 2}$

$R(10) = (25-(10-5)^2)^{1/2} - 1 = \sqrt{25-25} - 1 = 0 - 1 \boxed{= -1}$

b) $(R+1)^2 + (F_{t-1}-5)^2 = 25 \rightarrow$ equation for a circle of radius 5




c) In a "state of nature," $R(F_{t-1}) = 0$

$R(F_{t-1}) = (25-(F_{t-1}-5)^2)^{1/2} - 1 = 0$

$(25-(F_{t-1}-5)^2)^{1/2} = 1$

$25-(F_{t-1}-5)^2 = 1$

$24 = (F_{t-1}-5)^2$

$\sqrt{24} = F_{t-1} - 5$

$\boxed{F_{t-1} = \sqrt{24} + 5} \doteq 9.899$

d) If the fishermen catch three fish, the graph of $R(F_{t-1})$ will be shifted down 3. We can tell from the graph that the new equilibrium quantity of fish will occur where $\boxed{F_{t-1} = 8.}$

e) At the maximum sustainable catch, $R(F_{t-1})$ is at a maximum

$\frac{dR}{dF_{t-1}} = \frac{1}{2}(25-(F_{t-1}-5)^2)^{-1/2} \cdot -2(F_{t-1}-5) = \frac{10-2F_{t-1}}{2\sqrt{25-(F_{t-1}-5)^2}} = 0$

$10 - 2F_{t-1} = 0 \qquad 10 = 2F_{t-1} \qquad \boxed{F_{t-1} = 5}$

f) The fishermen's catch of 6 would exceed the maximum sustainable catch of 5, causing a decline in the fish population that would eventually lead to its extinction.