Chapter 5 Problem Set

2. a. The $100,000 depreciation expense would have increased to $205,000 - $100,000 would be added as a result of the accelerated depreciation, and $5,000 would be added as a result of the new yearly capital depreciation ($50,000 over 10 years). Material expenses would have decreased $30,000 to $70,000, and labor would decrease $20,000 to $200,000. Reported profits would drop to $245,000; only $85,750 in taxes would be paid, so after-tax profits would decrease to $159,250; the addition to retained earnings would drop to $59,250.

(Attached is a copy of the new income statement.)

b. If the firm were to reclassify $20,000 of labor and materials as a capital expense, reported profits would increase by $18,000 ($20,000 from the labor/materials loss - $2,000 from the depreciation expense). In turn, the firm would have to pay a higher corporate tax ($5,200 higher). On the other hand, the higher reported profits would likely increase demand for the company's stock - and thus its price - and improve public perception of the company. Given that this reclassification is illegal, however (re: WorldCom), it would be an unwise move by the company. Good.

3. \( q = K^{1/2} L^{1/2} \)  
   \( K = \text{capital}, \ L = \text{hours of labor}, \ q = \text{hourly output} \)

   a. 1 machine, 16 workers \( \Rightarrow K = 1, \ L = 16 \)

   \( \text{output per hour} = \frac{1}{16} \cdot 16 = 1 \)  
   \( \frac{d(1)}{dL} = 0 \quad (K = \text{constant}) \)

   average product of labor = \( \frac{16}{16} = 1 \)

   marginal product of labor = \( \frac{16}{16} = 1 \)

   marginal product of capital = \( \frac{1}{2} \cdot 1 = \frac{1}{2} \)

   \( \text{MFC} = \frac{\text{dTC}}{\text{dL}} = \frac{\text{dTC}}{\text{dK}} \cdot \frac{\text{dK}}{\text{dL}} \)

   \( \frac{1}{2} \cdot \frac{1}{8} = \frac{1}{16} \)

   \( \text{MPP} = \frac{1}{2} \cdot \frac{1}{8} = \frac{1}{16} \)

b. If the production function is homogeneous of degree 1, \( q(pL, pK) = p q(L, K) \). Since \( L \) and \( K \) are both raised to the same power, the equation is homogeneous of degree 1. For example, if \( p = 2, \ 16 \cdot 16^{1/2} = 8 = 2(4) \). If \( p = 3, \ 13 \cdot 13^{1/2} = 12 = 3(4) \). If \( K \) and \( L \) were raised to different powers - \( K^{a} L^{b} \) for example - the equation would not be homogeneous of degree 1.

C. \( \text{wage} = \$11.00/\text{hr}, \ \text{machine} = \$20/\text{hr} \)

\( q = 10 = L^{1/4} (K^{1/4} = 1) \)

\( L = 100 \) \( \Rightarrow \) The firm would have to hire 100 workers.

\( \text{total cost} = 100(10) + 20 = \$1,020 \)

\( \text{ATC} = \frac{\text{TC}}{Q} = \frac{10020}{10} = 1002 \)

\( \text{AVC} = \frac{\text{TC} - \text{FC}}{Q} = \frac{10020 - 20}{10} = 900 \)
d. \( C(\theta, K) = K + \omega L^2 \)  
\[ q = K^{\frac{1}{2}} L^{\frac{1}{2}} \]  
\[ L^* = K \]  
\[ K = q^2 \]  

Since \( K = 1 \), \( L = q^2 \) 
\[ C(q, K) = q K + \omega q^2 \]  
\[ r = 20, \omega = 10 \]  
\[ C(q) = 20 + 10q^2 \]

\[ e. C(\theta) = 20 + 10q^2 \]  
\[ C'(\theta) = 20q \]  
\[ C'(10) = 20(10) = 200 \]

Average cost \( \frac{C(q)}{q} = \frac{20 + 10q^2}{q} = \frac{20}{q} + 10q \)  
\[ \text{ATC}(10) = \frac{20}{10} + 10(10) = 2 + 100 = 102 \]

Average variable cost \( \frac{C(q) - 20}{q} = \frac{10q^2}{q} = 10q \)  
\[ \text{AVC}(10) = 10(10) = 100 \]

5. \( p(q) = 20 - 2q, C(q) = 4q + q^2 \)

a. \( R(q) = q \cdot p(q) = q(20 - 2q) \)  
\[ R(q) = 20q - 2q^2 \]  
\[ \frac{dR}{dq} = 20 - 4q \]

b. \( \pi = R(q) - C(q) \) 
When \( \pi \) is maximized, \( \frac{d\pi}{dq} = 0 \)  
\[ 20 - 4q = 4 + 2q \]  
\[ 16 = 6q \]  
\[ q = \frac{16}{6} = \frac{8}{3} \]

\[ p(q) = 20 - 2q = 20 - 2(\frac{8}{3}) = \frac{60}{3} - \frac{16}{3} = \frac{44}{3} \approx 14.67 \]
\[ R(\frac{8}{3}) = 20(\frac{8}{3}) - 2(\frac{8}{3})^2 = \frac{256}{9} - \frac{64}{9} = \frac{192}{9} = 21.33 \]
\[ C(\frac{8}{3}) = 4(\frac{8}{3}) + (\frac{8}{3})^2 = 17.78 \]
\[ \pi(\frac{8}{3}) = R(\frac{8}{3}) - C(\frac{8}{3}) = 21.33 - 17.78 = 3.55 \]
\[ \text{MC} = C'(q) = 4 + 2q = \frac{4}{3} + \frac{16}{3} = 6.67 \]
\[ \text{ATC} = \frac{C(q)}{q} = \frac{4q + q^2}{q} = 4 + q \]
\[ \text{per-unit profit} = \frac{\pi}{q} = \frac{3.55}{\frac{8}{3}} = 1.55 \]
c. $p = \frac{412}{q}$

\[ \frac{\text{MC}}{\text{ATC}} = \frac{412}{14 + q} \]

\[ q = -\frac{10}{p} + 10 \]

\[ p(8) = 20 - 2q \]

\[ p - 10 = 2q \]

\[ q = -\frac{10}{p} + 10 \]

\[ \text{d. CS} = \text{shaded area on graph in (c)} \]

\[ = \frac{1}{2} \left( 20 - 2q - \frac{412}{q} \right) \cdot q \]

\[ = 20 \cdot \frac{q}{2} - \frac{210}{q} \]

\[ = \frac{16q^2 - 210}{2q} \]

\[ = \frac{16}{3} - \left( \frac{1}{3} \right) \]

\[ = \frac{16 \cdot \frac{2}{9}}{3} \cdot \frac{9}{q} = \frac{320}{q} = 16.67 \]

\[ \text{g. C}(q) = 16 + 4q + q^2 \]

\[ \text{a. ATC} = \frac{16 + 4q + q^2}{q} = \frac{16}{q} + 4 + q \]

\[ \text{ATC}_{10} = \frac{16}{10} + 4 + 10 = \frac{14}{5} + 24 = \frac{154}{5} = 30.8 \]

\[ \text{AVC} = \frac{4q + 4}{q} = 4 + \frac{4}{q} \]

\[ \text{AVC}_{10} = 4 + \frac{4}{10} = 4.4 \]

\[ \text{MC} = C'(q) = 4 + 2q \]

\[ C'(20) = 4 + 2(20) = 44 \]
b. \( p = 20 \) in a competitive market
\[ R'(q) = C'(q) \]
In a competitive market, \( p = MR = 20 \)
\[ C'(q) = 4 + 2q = 20 \]
\[ 2q = 16 \]
\[ q = 8 \]
\[ q \geq 8 \]

C. \( p = 10 \) in the short term
\[ R'(q) = p = 10 \]
\[ R'(q) = C'(q) = 10 \]
\[ C'(q) = 4 + 2q = 10 \]
\[ 2q = 6 \]
\[ q = 3 \]
Income statement of the Fly-by-Nite Aircraft Company, 2001

January 1, 2001 to December 31, 2001 (all figures in $1,000,000)

Net sales 800

Less:
Cost of manufacturing
  Materials 200
  Labor 250
  Depreciation expense 100

  Less inventory increase (100)

Cost of goods sold (450)

Gross margin 350
  Less selling cost 30
  Interest expense 20

(50)

Profits 300
  Less corporate profit tax (105)

Profits after taxes 195
  Less dividends (100)

Addition to retained earnings 95

Income statement of the Fly-by-Nite Aircraft Company, 2001 (a)

January 1, 2001 to December 31, 2001 (all figures in $1,000,000)

Net sales 800

Less:
Cost of manufacturing
  Materials 170
  Labor 230
  Depreciation expense 205

  Less inventory increase (100)

Cost of goods sold (505)

Gross margin 295
  Less selling cost 30
  Interest expense 20

(50)

Profits 245
  Less corporate profit tax (86)

Profits after taxes 159
  Less dividends (100)

Addition to retained earnings 59

(Note: all figures are rounded to the nearest $1,000)
Income statement of the Fly-by-Nite Aircraft Company, 2001 (b)

January 1, 2001 to December 31, 2001 (all figures in $1,000,000)

Net sales 800

Less:
Cost of manufacturing
  Materials 190
  Labor 240
  Depreciation expense 102
  532
Less inventory increase (100)
Cost of goods sold (432)

Gross margin 368
Less selling cost 30
  Interest expense 20
  (50)

Profits 318
Less corporate profit tax (111)
Profits after taxes 207
Less dividends (100)
Addition to retained earnings 107

(Note: all figures are rounded to the nearest $1,000)