Economics 110, Section 2 Due: Noon, September 13, 2006

Revised 9/13/06 Name ____________________

Problems – Chapter 2

Note: Please show your work. Be neat!
You should deposit your completed problem set in the E110 Section 2 slot around the corner to the left of the Econ Office (1st 123 PAC) by 12:00 noon, on Wednesday, September **13**th.

Read carefully Chapter 2 (Section 2.4 is optional) before answering these questions.

1. A farmer can produce goods x (eXlax) and y (Yams) on his farm. The quantity of y that can be produce is negatively related to the output of x. Specifically, our farmer can produce no more than six units of x. Further, the output of y that can be produced is less the more x that is produced. Also, output cannot be negative. Specifically, outputs must satisfy the following inequalities

$$0 \le y \le 36 - x^2, \quad 0 \le x \le 6$$

Or to put it another way, the production transformation function is

$$y = 36 - x^2, \quad 0 \le x \le 6$$

a. Determine dy/dx when x = 3; find dy/dx as a function of x.

b. What is the marginal rate of transformation when x = 3?

$MRT_{y\text{ into }x} = -dy/dx =$ ____ ; [1] $MRT_{x\text{ into }y} =$ ____

c. Suppose that the price of x is $p_x = \$8$ and the price of y is $p_y = \$2$. Determine the quantities of x and y that should be produced in order to maximize profits, π, assuming that there are no production costs so total revenue equals profits:

$$\pi(x,y) = 8x + 2y$$

Hint: First substitute the production transformation function into the profit function to obtain profit as a function of only x: $\pi^*(x) = 8x + 2(36-x^2)$.

d. What is the marginal rate of transformation (y into x) when the firm is maximizing profit?
Is our firm's $MRT_{\mathbf{y\ into\ x}} = \mathbf{p_x/p_y}$ when profits are maximized?

> *Proposition:* If a firm producing X and Y with zero production costs is maximizing profits then $p_x/p_y = MRT_{\mathbf{y\ into\ x}}$
> (See tangency point e on Figure 2.3, page32)
> *Proof:* Let T(x) = y denote the production transformation curve.
> Then substituting T(x) for y into $\pi(x,y) = p_x x + p_y y$ yields $\pi^*(x) = p_x x + p_y T(x)$.
> Profit maximizing requires $d\pi^*/dx = p_x + p_y dy/dx = 0$ or $p_x/p_y = -dy/dx = \mathbf{MRT_{y\ into\ x}}$

e. Suppose that inflation were to cause the price of x and the price of y to both double, $p_x = \$16$ and $p_y = \$4$. How would this affect the output of x and y?

f. Determine the supply function for x showing the quantity of x that will be produced as a function of p_x given that that $p_y = \$2$, and assuming that the

[1] See my Email of 9/7/06 about the distinction between the two MRT concepts

objective is to maximize profits (there are no production costs). Then determine, in general, $x(p_x, p_y)$ the supply of **x** as a function of p_x and p_y.

2. The 100 farmers in England each have the same production transformation function of question 1: $y = 36 - x^2$. The 200 farmers in Portugal also have the same production function.
   a. Each farmer in England produces x = 2 and y = 32. Commodity y sells at a price of £2 (2 pounds). What price of **x** must prevail in England if this output is being produced by profit (= revenue, as there are no production costs) maximizing farmers? [Hint: First calculate the $MRT_{\mathbf{y\ into\ x}}$**]**
   b. The 200 profit-maximizing farmers in Portugal are each producing x = 5. How much y are they producing?
   Commodity y sells for €1 (Euro). What must be the price of x in Portugal?
   c. England and Portugal together are producing 1200 units of x. How much y are they producing? Fill in the blanks.

		Each farmer		Number of farmers	Each country	
Original		x output	y output		x output	y output
	England	2	_____	100	200	_____
	Portugal	5	_____	200	1,000	_____
Total			_____	_____	1,200	_____

   d. Reallocate the production of the goods between the two countries so that the two countries combined still produce 1200 units of x but more y!

   e. You have inherited a boat and £40[2] from your late Uncle Rich. Consider the following alternative arbitrage operations: **(Assume transportation costs are neglible)**

   Arbitrage Operation A: Buy **£40** worth of good y in England, ship it to Portugal, trade it for good x in Portugal and bring the x back to **sell in** England.

   Arbitrage Operation B: Purchase **£40 worth** of good x in England, ship it to Portugal, exchange it in Portugal for good y, and ship the y back to **sell in** England.

   Which Arbitrage Operation will yield you the most profit? How many pounds?

Class Discussion Questions:

1. Does your arbitrage operation benefit the citizens of England and/or Portugal? Does it hurt anyone? Explain
2. If a large number of traders attempt to profit in this way, how will prices change and how will the opportunity for profit be affected?

Honors Option Questions: # 7 and #8, page 56 of the text. (If you choose to try the Honors option question, you must work on your own without assistance from the TA or the instructor)

2 The symbol £ denotes the British Pound, which initially was equal in value to a troy ounce of silver.