Problems – Chapter 2

Read carefully Chapter 2 (Section 2.4 is optional) before answering these questions.

1. A farmer can produce goods x (eXlax) and y (Yams) on his farm. The quantity of y that can be produce is negatively related to the output of x. Specifically, our farmer can produce no more than six units of x. Further, the output of y that can be produced is less the more x that is produced. Also, output cannot be negative. Specifically, outputs must satisfy the following inequalities

\[ 0 \leq y \leq 36 - x^2, \ 0 \leq x \leq 6 \]

Or to put it another way, the production transformation function is

\[ y = 36 - x^2, \ 0 \leq x \leq 6 \]

a. Determine \( dy/dx \) when \( x = 3 \); find \( dy/dx \) as a function of \( x \).

b. What is the marginal rate of transformation when \( x = 3 \)?

\[ \text{MRT}_{y \text{ into } x} = -\frac{dy}{dx} = \quad ; \text{MRT}_{x \text{ into } y} = \quad \]

c. Suppose that the price of x is \( p_x = $8 \) and the price of y is \( p_y = $2 \). Determine the quantities of x and y that should be produced in order to maximize profits, \( \pi \), assuming that there are no production costs so total revenue equals profits:

\[ \pi(x,y) = 8x + 2y \]

Hint: First substitute the production transformation function into the profit function to obtain profit as a function of only x: \( \pi^*(x) = 8x + 2(36-x^2) \).

d. What is the marginal rate of transformation (y into x) when the firm is maximizing profit?

Is our firm’s \( \text{MRT}_{y \text{ into } x} = \frac{p_x}{p_y} \) when profits are maximized?

\textbf{Proposition:} If a firm producing X and Y with zero production costs is maximizing profits then \( \frac{p_x}{p_y} = \text{MRT}_{y \text{ into } x} \)

(See tangency point e on Figure 2.3, page32)

\textbf{Proof:} Let \( T(x) = y \) denote the production transformation curve.

Then substituting \( T(x) \) for y into \( \pi(x,y) = p_x x + p_y y \) yields \( \pi^*(x) = p_x x + p_y T(x) \).

Profit maximizing requires \( d\pi^*/dx = p_x + p_y dy/dx = 0 \) or \( p_x/p_y = -dy/dx = \text{MRT}_{y \text{ into } x} \)

e. Suppose that inflation were to cause the price of x and the price of y to both double, \( p_x = $16 \) and \( p_y = $4 \). How would this affect the output of x and y?

f. Determine the supply function for x showing the quantity of x that will be produced as a function of \( p_x \) given that that \( p_y = $2 \), and assuming that the

---

\(^1\) See my Email of 9/7/06 about the distinction between the two MRT concepts
objective is to maximize profits (there are no production costs). Then determine, in general, \( x(p_x, p_y) \) the supply of \( x \) as a function of \( p_x \) and \( p_y \).

2. The 100 farmers in England each have the same production transformation function of question 1: \( y = 36 - x^2 \). The 200 farmers in Portugal also have the same production function.

a. Each farmer in England produces \( x = 2 \) and \( y = 32 \). Commodity \( y \) sells at a price of £2 (2 pounds). What price of \( x \) must prevail in England if this output is being produced by profit (= revenue, as there are no production costs) maximizing farmers? [Hint: First calculate the MRT \( y \text{ into } x \)]

b. The 200 profit-maximizing farmers in Portugal are each producing \( x = 5 \). How much \( y \) are they producing? Commodity \( y \) sells for €1 (Euro). What must be the price of \( x \) in Portugal?

c. England and Portugal together are producing 1200 units of \( x \). How much \( y \) are they producing? Fill in the blanks.

<table>
<thead>
<tr>
<th>Each farmer</th>
<th>Number of farmers</th>
<th>Each country</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>x output</td>
<td>y output</td>
</tr>
<tr>
<td>England</td>
<td>2</td>
<td>_____</td>
</tr>
<tr>
<td>Portugal</td>
<td>5</td>
<td>_____</td>
</tr>
<tr>
<td>Total</td>
<td>_____</td>
<td>_____</td>
</tr>
</tbody>
</table>

d. Reallocate the production of the goods between the two countries so that the two countries combined still produce 1200 units of \( x \) but more \( y \)!

e. You have inherited a boat and £40\(^2\) from your late Uncle Rich. Consider the following alternative arbitrage operations: (Assume transportation costs are negligible)

*Arbitrage Operation A:* Buy £40 worth of good \( y \) in England, ship it to Portugal, trade it for good \( x \) in Portugal and bring the \( x \) back to sell in England.

*Arbitrage Operation B:* Purchase £40 worth of good \( x \) in England, ship it to Portugal, exchange it in Portugal for good \( y \), and ship the \( y \) back to sell in England.

Which Arbitrage Operation will yield you the most profit? How many pounds?

Class Discussion Questions:

1. Does your arbitrage operation benefit the citizens of England and/or Portugal? Does it hurt anyone? Explain

2. If a large number of traders attempt to profit in this way, how will prices change and how will the opportunity for profit be affected?

Honors Option Questions: #7 and #8, page 56 of the text. (If you choose to try the Honors option question, you must work on your own without assistance from the TA or the instructor)

---

\(^{2}\) The symbol £ denotes the British Pound, which initially was equal in value to a troy ounce of silver.