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Growth and Development

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12.1 Introduction

Why some nations grow while others stagnate is a question that has captured the interests of generations of economists. And economists have not always been optimistic about the long run destiny of nations. The classical school of economists, led by Adam Smith and David Ricardo, worried that

Table 12.1. International comparisons of growth of GDP per capita.

GDP per capita (1990 $)						
	1700	1820	1870	1950	1973	1998
China	600	600	530	439	839	3,117
Germany	894	1,058	1,821	3,881	11,966	17,799
Italy	2,100	1,921	2,753	5,996	13,082	20,224
Japan	520	570	737	1,926	11,439	20,413
Mexico	568	759	674	2,365	4,845	6,655
United Kingdom	1,250	1,707	3,191	6,907	12,022	18,714
United States	527	1,257	2,445	9,561	16,689	27,331
Africa	400	418	444	852	1,365	1,368
World	615	667	867	2,114	4,104	5,709
GDP as a percent of U.S. GDP in 1998						
	1700	1820	1870	1950	1973	1998
China	2%	2%	2%	2%	3%	11%
Germany	3%	4%	7%	14%	44%	65%
Italy	8%	7%	10%	22%	48%	74%
Japan	2%	2%	3%	7%	42%	75%
Mexico	2%	3%	2%	9%	18%	24%
United Kingdom	5%	6%	12%	25%	44%	68%
United States	2%	5%	9%	35%	61%	**100%**
Africa	1%	2%	2%	3%	5%	5%
World	2%	2%	3%	8%	15%	21%

Source: Angus Maddison, *The World Economy: A Millennial Perspective*, Table 5-21, p. 264.

a maturing economy will inevitably approach a "stationary state" characterized by a bare subsistence standard of living and zero economic growth. No wonder economics has been called "the dismal science."

The gloomy predictions of the classical economists were wrong! As is clear from the data presented in Chapter 1.5.1, growth has been the big economic story of the last two centuries. The data on Table 12.1 tell more of the growth story. The evidence, which is measured in international dollars of constant purchasing power so as to permit comparisons both among countries and over time, displays a mixed picture. Some nations have stagnated, but much of the world has enjoyed a dramatic increase in real income rather than the decline predicted by the classical school. In some countries, the average citizen's living standard has doubled and then doubled again in a single lifetime!

This chapter begins by presenting the "classical" argument concerning the inevitability of the stationary state. Then we shall construct a "neo-classical" growth model explaining how technological advance and capital accumulation can, under certain conditions, lead to a continuing

improvement of living standards for a growing population in spite of the law of diminishing returns. Later in this chapter we will also ask whether we can count on the market mechanism to allocate petroleum and other exhaustible resources appropriately over time or whether we risk squandering our limited resources to the detriment of future generations. We will also consider a simple model of "over-fishing."

12.2 Malthusian population dynamics

In 1798 the Reverend Thomas R. Malthus [1766–1834] anonymously published *An Essay on the Theory of Population.* Malthus warned the readers of his best seller that while the population tended to grow like a geometric series $(2, 4, 8, 16, \ldots)$, the food supply only grows arithmetically $(1, 2, 3, 4, \ldots)$. As a result, Malthus argued, population growth has an inevitable tendency to outstrip the world's food supply. Therefore, a deteriorating standard of living and widespread hunger are inevitable, unless moral restraint holds reproduction in check. Throughout the 19th and well into the 20th century, many economists, and indeed the public generally, worried about the pessimistic Malthusian prediction.

The Law of Diminishing Returns (recall Chapter 5.4.2) provides a link between population growth and the food supply that supports the prediction of Malthus. Since the amount of land in the world is a fixed resource (Holland, thanks to its dikes, being a notable exception), the law of diminishing returns implies that, if the world's population keeps growing, the supply of food available per worker must eventually decline, as illustrated on Figure 12.1. Here is a "scientific" case for population control.

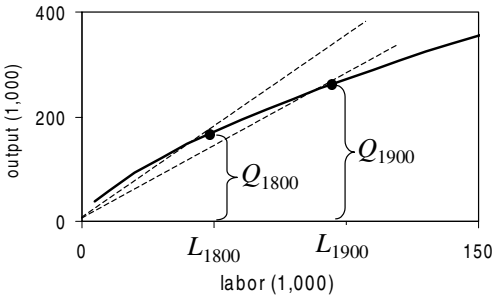

Fig. 12.1. Diminishing returns
The increase in the supply of labor leads to greater output in Econoland. But because of the law of diminishing returns, the increase in the labor supply, other things being equal, leads to a decline in the ratio Q/L, or output per capita.

While the prediction of Malthus appears to this day to be all too true in many sectors of the globe, the evidence on per capita output growth presented in Chapter 1.5.1 and on Table 12.1 makes it clear that the law is far from a universal truth. There are many hungry in the world, but on average the world's population is better fed than at anytime in history — contrary to Malthus, the food supply has grown more rapidly than the population.

12.3 A classical growth model, simplified

Let us begin by considering a model that captures certain essential features of the classical analysis of the growth process as expounded by Adam Smith, David Ricardo and their followers. This model predicts, as did the classical economists, that the law of diminishing returns means that there will be a gradual decline in living standards. The economy will inevitably mature into a *stationary state* characterized by a stable population, zero output growth and a subsistence standard of living. Obviously, this prediction went wrong, but much is to be learned from finding the source of the prediction error. The discussion will set the stage for the subsequent construction of a more optimistic model of the growth process.

The subsistence theory of wages assumes that the rate of growth of the labor force depends on how the average real wage rate w_g compares with the subsistence wage w^s. Here the subscript g indicates the generation; i.e., we use as the unit for recording time the number of years required for a generation to replicate itself — perhaps 25 years equals one generation. If the wage w_g that parents of generation g receive is above the subsistence wage w^s that is required for subsistence, they will have more children and, as a result, the next generation will be larger and so will the work force. If the wage rate is below the *subsistence wage*, the population and hence the workforce will shrink. The conjecture that it is the gap between the wage w_g that workers of each generation actually receive and the subsistence wage w^s that determines the rate of population growth is captured by the equation

$$\frac{L_{g+1} - L_g}{L_g} = \left(\frac{w_g}{w^s}\right)^{\alpha} - 1\,, \quad \alpha > 0\,, \tag{1}$$

or

$$L_{g+1} = \left(\frac{w_g}{w^s}\right)^{\alpha} L_g\,, \quad \alpha > 0\,. \tag{2}$$

Let us also suppose that the production process involves only two inputs, land which is in fixed supply, and labor. To be concrete, suppose that a function of Cobb-Douglas form determines output for generation g,

$$Q_g = \rho L_g^{\lambda} R^{1-\lambda}\,, \quad 0 < \lambda < 1\,, \tag{3}$$

where L_g is the labor force and R (for resources) measures the fixed supply of land. Assuming that the market for labor is competitive, workers will be hired up to the point where the marginal product of labor is equal to the real wage; as was explained in Chapter 7.3; i.e.,

$$w_g = \frac{\partial Q_g}{\partial L_g} = \lambda \rho L_g^{\lambda-1} R^{1-\lambda} = \frac{\lambda Q_g}{L_g}\,. \tag{4}$$

Suppose we are given the subsistence wage w^s, the values of the parameters of the system and the initial size of the labor force for a particular generation, say L_0. Then it is possible to determine the future time path of the labor force. First we calculate Q_0 using equation (3). Then (4) yields w_0 so that we can finally calculate L_1 from L_0 using equation (2). Once we have L_1 we can repeat the procedure to calculate Q_1 and w_1 and then L_2 and so on into the indefinite future. Figure 12.2 indicates what happens. Note that both population and output will continue to grow, but at

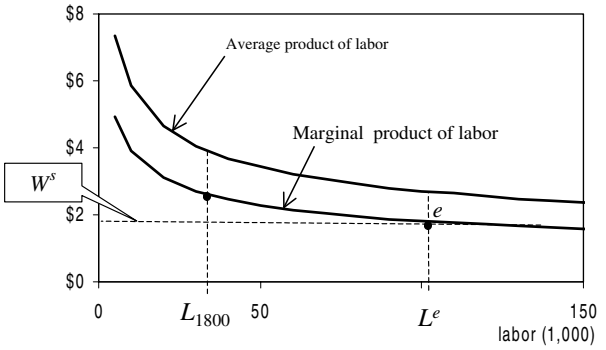

Fig. 12.2. Diminishing returns and the classical stationary state
Because of diminishing returns, the average and marginal products of labor are decreasing functions of the labor input. Since the supply of labor was initially 30,000 at L_{1800}, the wage will equal the marginal product of labor of approximately \$3.33. Because this wage is above subsistence wage $w^s = \$2.00$, the labor force expands, pushing down both the average and marginal product of labor. The labor force will continue to grow as long as $w > w^s$, which pushes us toward an unhappy equilibrium at point e where the wage is at the subsistence level of \$2 but output per worker is \$3.00.

slower and slower rates as the system gradually approaches the stationary equilibrium labor force L_e with wage w^s in the limit. Since output grows less rapidly than labor, thanks to the law of diminishing returns, the wage rate will inevitably be driven down to its subsistence level. The long-run equilibrium for this model is not a pretty sight.

Prediction fallacies

The pessimistic predictions of Malthus and the classical economists have been contradicted by history. Three factors account for the failure of this model to predict what happened.

1. The model fails to capture the upward shift in worker productivity brought about by the twin contributions of invention and capital accumulation. The steam engine, the internal combustion engine, electric power and now computers are but four of the inventions that have made decisive contributions to greater worker productivity. Malthus and the classical economists grossly underestimated the contributions of investment and technological progress.
2. Capital accumulation, made possible by thrift or abstinence from consumption, contributed to increased worker productivity.

   How technological progress can offset the effect of diminishing returns is illustrated by the total product curves plotted on Figure 12.3. The growth in population allows more and more workers to be employed with the passage of time. Output per capita would decline if the total project curve had remained unchanged at its 1800 level. But over time the development of better production techniques and the accumulation of physical capital shifted the total product curve upwards, thereby enabling a gradual increase in output per capita and rising living standards.
3. The assumption that the rate of population growth is governed by the gap between the wage received by workers and the subsistence wage, a critical assumption invoked to explain why the wage would be driven to the subsistence level, proved to be grossly inaccurate.

The law of diminishing returns may hold, given the technology and fixed supplies of natural resources and productive capital, but the historical record is clear: The unanticipated pace of technological advance coupled with substantial investment in productive capital has enabled output in the majority of countries to outstrip the growth of the workforce and has provided welcome increases in average living standards.

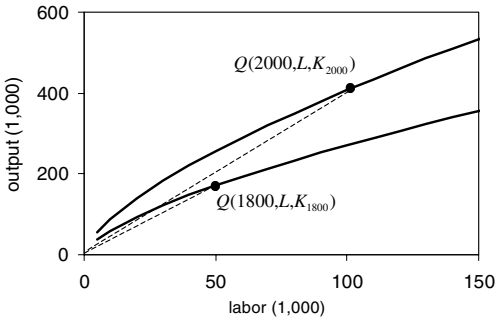

Fig. 12.3. Technological progress offsets the law of diminishing returns
Technological improvements and capital accumulation caused an upward shift in the total product curve from 1800 to 2000 that more than offset the tendency of the growing labor force to reduce the average and marginal products of labor. As a result, real wages increased rather than declining toward the subsistence level.

12.4 Growth accounting — The sources of economic growth

Output grows as a result of an increase in hours of work. It also increases if workers are able to work more efficiently because they are equipped with more capital equipment. And it will also grow because of the development of better production techniques. But how can we determine the relative importance of each of these factors in explaining economic growth? The question is of considerable policy interest. Definitely establishing that technological change plays the major role would imply that an increase in taxes to subsidize research and development might make a decisive contribution to greater growth. But if it turns out that investment is the decisive factor, then higher taxes, by discouraging thrift and investment, might slow the pace of economic development. If investment is the critical determinant, rapid growth might be encouraged by government subsidies and tax benefits promoting private investment spending.

Unfortunately, the contribution of technological improvements is hard to quantify. We have real GDP as a measure of total output and we can count the number of hours worked during the year, but how can we measure the contribution to worker productivity of the internal combustion engine, the assembly line, and the computer revolution? Almost a half century ago Nobel Laureate Robert Solow pioneered a procedure for estimating the contribution of technological progress that is still in use today.[1]

[1]Robert Solow, "Technical change and the aggregate production function," *Review of Economics and Statistics*, 1957.

Let us suppose that the aggregate output of the economy at time t is determined by the production function

$$Q(t) = f(t, L(t), K(t)), \tag{5}$$

where $L(t)$ is the number of labor hours and $K(t)$ is the stock of capital in year t. We include t as an argument in the production function to indicate that output would increase with the passage of time as a result of technological change, even if K and L were to remain constant.

Differentiating (5) with respect to time yields the total derivative:

$$\frac{dQ(t)}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial L}\frac{dL}{dt} + \frac{\partial f}{\partial K}\frac{dK}{dt}. \tag{6}$$

This total derivative says that the change in output is the sum of three components: first is the upward shift due to technological advance, second is the increase in output due to the growth of labor and third is the increase due to the availability of additional capital equipment. Note that the contribution of growing labor to increased output is the increase in labor dL/dt times the marginal productivity of labor.

We can manipulate (6) to obtain

$$\frac{dQ(t)/dt}{Q} = \frac{\partial f/\partial t}{Q} + \frac{\partial f}{\partial L}\frac{L}{Q}\frac{dL/dt}{L} + \frac{\partial f}{\partial K}\frac{K}{Q}\frac{dK/dt}{K}. \tag{7}$$

This simplifies to

$$q = \rho + \frac{\partial f}{\partial L}\frac{L}{Q}n + \frac{\partial f}{\partial K}\frac{K}{Q}k, \tag{8}$$

where $q = \frac{dQ(t)/dt}{Q}$ is the rate of growth of output, $n = \frac{dL/dt}{L}$ is the rate of growth of the labor-force, $k = \frac{dK/dt}{K}$ is the rate of growth of the capital stock, and $\rho = \frac{\partial f/\partial t}{Q}$ is the contribution of technological change. Thus we have decomposed the rate of growth of output into three components: the first is the contribution of technological change, the second is the contribution of labor-force growth, and the third is the contribution of the increased stock of capital.

In order to make the task of estimating the contribution of technological change manageable, Solow made two fundamental assumptions: He assumed that markets are competitive. He also assumed that the production function is homogeneous of degree 1 in capital and labor, which means that given the technology, doubling the quantities of both capital and labor will double output. Recall, once more that under competition the real wage

equals the marginal product of labor; therefore, the coefficient of labor force growth in (8) is $wL/pQ = \lambda_L$, which is the concept of labor's share discussed in Chapter 7.3.3. By a parallel argument, the coefficient of the rate of growth in the capital stock k is equal to $rK/pQ = \lambda_K$, or capital's share, where r is the rental cost of capital. Now the assumption that the production function is homogeneous of degree 1 in capital and labor implies that $\lambda_L + \lambda_K = 1$.[2] If we follow Solow in invoking these two assumptions we have for the rate of output growth:

$$q = \rho + \lambda_L n + (1 - \lambda_L)k\,. \tag{9}$$

The rate of growth of per capita output, Q/L,[3] is

$$q - n = \rho + (1 - \lambda_L)(k - n)\,. \tag{10}$$

Solow recognized that the only unobservable in equation (9) is the rate of technological progress, ρ. So he calculated by subtraction what has ever since been known as the *Solow residual*:

$$\rho = q - \lambda_L n - (1 - \lambda_L)k\,. \tag{11}$$

Solow's residual estimate equals what is left over after the contributions of labor and capital growth are subtracted from the rate of growth of output.

Solow's Estimates

Applying residual equation (11) to annual data on q, n k and λ_L covering the period 1909 to 1949, Solow reported that ρ was about 1.2% per annum from 1909 to 1929 and about 1.9% per annum from 1929 to 1949. He concluded that about 7/8ths of the 80% increase in output per hour of work over the 40 year period was due to technological improvement and only 1/8th to an increase in the capital/labor ratio.

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[2]Homogeneity of degree 1 in capital and labor means that, given the level of technological development at any point of time t, if we double labor and capital we will double output. More precisely, for any coefficient $\varphi, \varphi Q = f(t, \varphi L, \varphi K)$; differentiating both sides of the equality with respect to φ yields $Q = (df/dL)L + (df/dK)K$,which is an example of Euler's Theorem. Dividing by Q gives us $1 = \lambda_L + \lambda_K$, as required.

[3]To see why we just subtract n to get the rate of per capita output growth, note that $\ln Q/L = \ln Q - \ln L$. Hence, differentiating both sides with respect to t yields

$$\frac{d(Q/L)/dt}{Q/L} = \frac{dQ/dt}{Q} - \frac{dL/dt}{L} = q - n\,.$$

Productivity Slowdown/Productivity Spurt

Productivity growth is not a smooth and predictable process, as can be seen from the top row of Table 12.2, which summarizes evidence developed by Dale W. Jorgenson and Kevin J. Sitroh.[4] From the end of World War II until about 1973, productivity growth in the United States took place at a remarkable clip. But around 1973 the economy floundered in what is known as the "slowdown in the rate of productivity growth," or simply the "productivity slowdown." Because of this productivity slowdown, output per hour of work grew at a much slower rate for the next two decades than it had in the preceding quarter century. Starting around 1995, there was a substantial spurt in productivity growth.

The differences in terms of annual percentages are not large. But life would be so much brighter if there had not been a slowdown in the rate of productivity growth.

1. A simple exercise in counterfactual history shows that the slowdown in productivity growth resulted in a substantial loss of output. Suppose that from 1973 to 1998 the rate of growth in output per hour of work had remained at the earlier 2.948% per annum clip reported in the first column of the table. Then by the simple equation for compound interest, in 1998 output per hour of work would have been $(1.02948)^{98-73} = 2.07$ times what it was in 1973. Instead, because of the slowdown, output per hour of work grew to 1.46 times its level in 1973.[5] Or to put it another way, if there had been no slowdown in the rate of productivity growth, output in 1998 would have been $2.07/1.46 = 1.41$ times its actual level of 1998, given the number of hours worked. With the same work effort, 41% more would have been produced!
2. More rapid productivity growth would have substantially reduced inflationary pressure during the last quarter of the 20th century. The analysis of Chapter 11.3.4 suggests that inflation would have been less of a problem and the natural unemployment rate (or NAIRU) would have been lower if productivity had been growing more rapidly. It is fair to say that the productivity spurt in the late 1990s encouraged Fed Chairman Alan Greenspan to allow the unemployment rate to drop to 4% without imposing substantial monetary constraint.

[4]Dale W. Jorgenson and Kevin J. Stiroh, "Raising the speed limit: U.S. economic growth in the information age." *Brookings Papers on Economic Activity*, I:2000, p. 151.

[5]We have $1.02948^{(98-73)} = 2.067$ and $1.01437^{(90-73)} \times 1.01366^{(95-90)} \times 1.02271^{(98-95)} = 1.46$

Table 12.2. Sources of U.S. labor productivity growth.

	(percent per annum)			
	1959–1973	1973–1990	1990–1995	1995–1998
Growth in labor productivity (Y/H)	2.948	1.437	1.366	2.271
Components:				
Capital deepening (K/L)	1.492	0.908	0.637	1.131
Labor quality	0.447	0.200	0.370	0.253
Total factor productivity	1.009	0.330	0.358	0.987

Source: Jorgenson and Stiroh, p. 151 (see text)

The bottom section of Table 12.2 breaks the growth in labor productivity into three components. According to Jorgenson and Stiroh, capital deepening, the increase in capital per worker, accounted for at least half of the increase in output per worker in each of the time periods recorded on the table. Jorgenson and Stiroh show that improvement in labor quality has been a contributing if somewhat erratic factor in the growth process. Growth in total factor productivity constitutes the remaining source of productivity growth. The authors report that information technology — computer hardware, software and communications — made a significant contribution to the growth in productivity in the last decade of the 20th century.

12.5 A neo-classical model of the growth process

What determines in the long run whether an economy will grow or decay? What determines the rate of growth? And why do some countries remain dormant while others take off into self-sustained growth. We will consider a pioneering contribution toward the resolution of such questions that is provided by the neo-classical model of economic growth developed in the 1950s by Robert Solow.[6] In order to focus on the essential issues of the growth process, we shall assume that technological change takes place at a constant rate. We will also assume that the labor force will grow at a

[6]Robert M. Solow, "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics*, February, 1956. The model presented here differs from Solow's in several respects. In particular, Solow did not restrict the production function to be of Cobb-Douglas form, but he did require constant returns to scale in labor and capital. There are no fixed resources in the original Solow model. Also, the analysis here is further simplified by using discrete rather than continuous time.

constant rate forever more. To further simplify, the simple model presented here leaves out both international trade and the role of government.

Notation: Lower case letters will denote rates of growth. For example, $q_t = \ln Q_t - \ln Q_{t-1} \doteq (Q_t - Q_{t-1})/Q_t$, where Q_t is Net Domestic Product in period t.

12.5.1 *Assumptions*

Let us assume that output Q_t is a function of labor L_t, capital K_t, and land R, where the level of output at any point of time is determined by the following production function:

$$Q_t = \alpha(1+\rho)^t L_t^{\lambda} K_t^{\lambda'} R^{1-\lambda-\lambda'} . \tag{12}$$

This elaborates on the Cobb-Douglas production function (equation (6) of Chapter 5) in two fundamental respects: First, it includes R for resources in fixed supply, such as land, as an additional input. With $\lambda + \lambda' < 1$, the function is not homogeneous of degree one in capital and labor: we have diminishing returns to scale in the two variable inputs, implying that a doubling of labor and capital would not double output. Second, technological progress is captured by the term $(1+\rho)^t$.[7] With $\rho > 0$, this means that if L_t, K_t, and R were to remain unchanged output would still grow with the passage of time because of improving techniques of production.

It is also assumed, for simplicity, that the population grows at constant rate n:

$$N_t = N_0(1+n)^t . \tag{13}$$

Further, a constant portion γ of the population is employed. Presumably, the labor force participation rate is constant and the employed proportion of the labor force does not vary, either because of the economy's natural self-recuperating powers or because the central bankers succeed in keeping the economy moving along its full-employment growth path. Therefore, the labor supply grows at rate n:

$$L_t = \gamma N_t = \gamma N_0(1+n)^t = L_o(1+n)^t . \tag{14}$$

[7] Readers familiar with elementary differential equations may prefer to work in continuous rather than discrete time, substituting $e^{\rho t}$ for $(1+\rho)^t$ in equation (12) and e^{nt} for $(1+n)^t$ in equations (13) and (14). See also footnote 8.

In addition, suppose that a constant fraction s of output is saved. Then consumption is $C_t = (1 - s)Q_t$. Since there is no government or foreign trade, $\mathrm{Q}_t = C_t + I_t$ and we have net investment

$$I_t = sQ_t\,. \tag{15}$$

12.5.2 *Analysis*

As a first step toward determining the laws of motion for this dynamic model, we ask whether output can grow at some constant exponential rate, call it q^e. To find out, let us first take logs to the base e of (12), with the approximation $\ln(1 + \rho) = \rho$:[8]

$$\ln Q_t = \ln\alpha + (1 - \lambda - \lambda')\ln R + t\rho + \lambda\ln\gamma + \lambda\ln N_t + \lambda'\ln L_t\,. \tag{16}$$

This equation holds for all t, including $t - 1$:

$$\ln Q_{t-1} = \ln\alpha + (1 - \lambda - \lambda')\ln R + (t-1)\rho + \lambda\ln\gamma + \lambda\ln N_{t-1} + \lambda'\ln L_{t-1}\,. \tag{17}$$

Subtracting equation (17) from (16) yields

$$\ln Q_t - \ln Q_{t-1} = \rho + \lambda(\ln L_t - \ln L_{t-1}) + \lambda'(\ln K_t - \ln K_{t-1})\,. \tag{18}$$

Invoking the approximation that the difference in the logs of a variable is its rate of change [e.g., $\ln Q_t - \ln Q_{t-1} \doteq (Q_t - Q_{t-1})/Q_{t-1} = q$], we have:

$$q = \rho + \lambda n + \lambda' k\,, \tag{19}$$

where n is the constant rate of growth of the labor force and k is the rate of growth of the capital stock. This equation says that if output is to grow at a constant rate q^e then k, the rate of growth of the capital stock, must also be constant. More than this, from (15) we have

$$sQ_t/K_t = I_t/K_t = k\,. \tag{20}$$

This means that the capital stock can grow at a constant rate k only if the output capital ratio, Q_t/K_t is constant, but that requires that Q_t and K_t grow at the same rate; i.e. $k = q^e$ if output grows at a constant rate. To find q^e, substitute it for q and k in (19) to obtain:

[8]For example, if $\rho = 3\%$, $\ln(1 + \rho) \doteq 2.95588\%$ using the approximation discussed in Chapter 8.4.2. Alternatively, working in continuous time, as mentioned in footnote 7, we can differentiate (16) with respect to t to obtain equation (19) directly.

$$q^e = \rho + \lambda n + \lambda' q^e = \frac{\rho + \lambda n}{1 - \lambda'} \,. \tag{21}$$

The rate of growth of output per capita is $q^e - n$. Per capita output will increase along the equilibrium growth path, output growing faster than the population, if and only if

$$q^e - n = \frac{\rho + \lambda n}{1 - \lambda'} - n > 0 \,, \quad \text{or} \quad \rho > (1 - \lambda - \lambda')n \,. \tag{22}$$

The properties of this growth equilibrium are clarified with the aid of Figure 12.4, which plots the output/capital ratio on the abscissa and rates of growth on the ordinate. The ray emanating from the origin denotes the equation $k = sQ_t/K_t$, from (20). The line labeled q is obtained by substituting $k = sQ_t/K_t$ into (19) to obtain

$$q = \rho + \lambda n + \lambda' s Q_t/K_t \,. \tag{23}$$

The intercept of the q line is $\rho + \lambda n > 0$. The slope of the k line is s, which means that it is steeper than the q line, whose slope is only $\lambda' s$. Hence the two lines must intercept. At the point where the q and k lines cross, marked e on the graph, we obviously have $q = k$. With output and capital both growing at the same rate there is no tendency for the (Q_t/K_t) ratio to change. This equilibrium point is characterized by $q^e = k^e = I/K$.

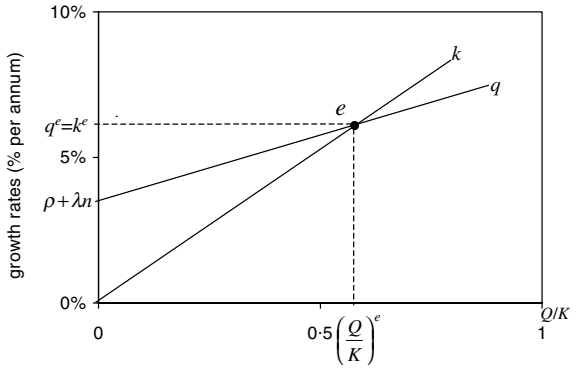

Fig. 12.4. Growth equilibrium
Both the growth rate of output (the q line) and the growth rate of the capital stock (the k line) depend on the Q/K ratio, plotted on the abscissa.

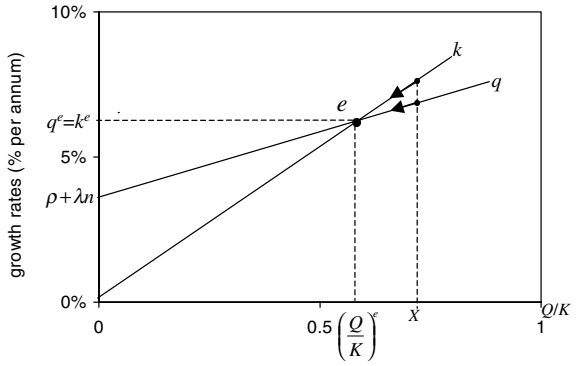

Fig. 12.5. Convergence to growth equilibrium
To see why the growth equilibrium at point e is stable, consider an emerging nation with a Q/K ratio above the equilibrium ratio, as at point X on the graph. Because the Q/K ratio is above the equilibrium value, K must be growing faster than Q (i.e., $k > q$), which means that the ratio Q/K must be falling toward the equilibrium value as indicated by the arrows on the graph.

Equations (20) and (21) imply that the corresponding equilibrium output/capital ratio is

$$\left(\frac{Q}{K}\right)^e = \frac{q^e}{s} = \frac{\rho + \lambda n}{s(1-\lambda')}\,. \tag{24}$$

This growth equilibrium is stable. To see why, consider a country that has yet to realize its full development potential. Suppose initially the output/capital ratio is $(\frac{Q}{K}) > (\frac{Q}{K})^e$, as illustrated by point X on Figure 12.5. Since its output/capital ratio is high, $q > q^e$; our country will be growing above its equilibrium rate, as can be seen from equation (23). But $k > q$ implies that the Q/K ratio is falling with the passage of time. Thus Q/K will approach its equilibrium value as a limit, as indicated by the arrows on the graph.

12.5.3 *Growth or stagnation?*

The time path by which a developing nation may gradually move toward a happy growth equilibrium is recorded on Table 12.3 and plotted on Figure 12.6. Our emerging nation has a capital stock growing much more rapidly than output, which means that the capital/output ratio is on the rise and yields rising output per worker. While the process of converging to equilibrium can be quite slow, the end result is a country cruising along its

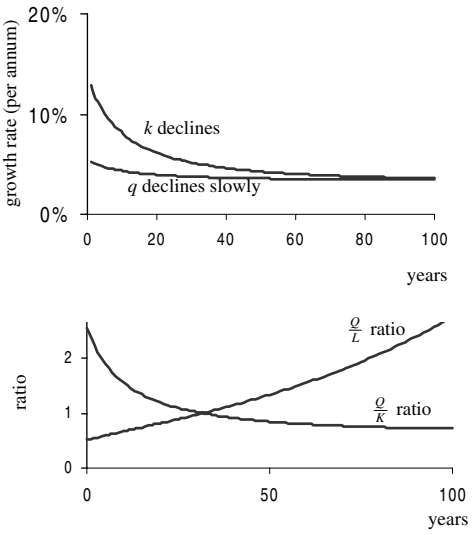

Fig. 12.6. Simulation #1: convergence to happy growth equilibrium
Top panel: The growth rates of output and of capital gradually decline toward the equilibrium growth rate.

Bottom panel: Output per worker, the Q/L ratio, grows rapidly as the economy moves into a more and more productive future. Since output per machine (Q/K) gradually declines while output per worker (Q/L) rises, the capital per worker ratio is increasing.

Parameter values: $\lambda = 0.65$, $\lambda' = 0.2$, $s = 5.0\%$, $n = 2\%$, $\rho = 1.4\%$,

Equilibrium values: $q_e = k_e = 3.3\%$; $(Q/K)^e = 0.67$; $q - n = 1.38\%$.

Table 12.3. Growth model simulation #1.

Parameter values: $\lambda = 0.65$, $\lambda' = 0.2$, $s = 5\%$, $n = 2\%$, $\rho = 1.4\%$, $K(0) = 20$, $N(0) = 100$, $R = 10$							
Year	N	K	Q	Q/K	q	k	Q/N
0	100	20	56	2.78			0.56
1	102	23	58	2.56	4.8%	13.9%	0.57
5	110	35	70	1.97	4.4%	10.4%	0.63
10	122	54	85	1.57	4.0%	8.2%	0.70
25	164	139	149	1.07	3.6%	5.5%	0.91
50	269	430	353	0.82	3.4%	4.1%	1.31
75	442	1,110	815	0.73	3.4%	3.7%	1.85
100	724	2,668	1,861	0.70	3.3%	3.5%	2.57
150	1,950	14,270	9,591	0.67	3.3%	3.4%	4.92
200	5,248	73,849	49,189	0.67	3.3%	3.3%	9.37

full-employment growth path with a constant rate of growth for both output and capital and a stable capital/output ratio, as specified by equations (21) and (24).

Figure 12.7 shows, for a different set of parameters, a most unhappy case in which the rate of growth of output is less than the rate of population growth, which means that the standard of living must inevitably decline! Comparison of the parameters with those of the earlier simulation reveals that this stagnant nation has a higher rate of population growth (n) coupled with a much less rapid pace of technological change (ρ).

Returning to equation (22), we find that improving this country's living standards would require a rate of technological change of at least 0.8% per annum, given the rapidly growing labor force coupled with the fact that certain resources R are in fixed supply means that the production function is subject to diminishing returns to scale in capital and labor,

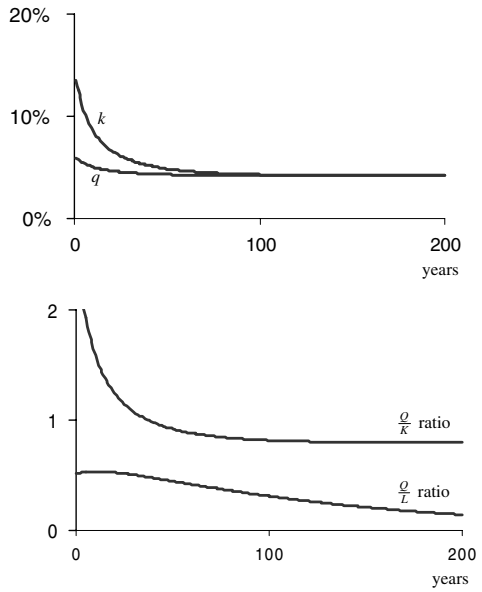

Fig. 12.7. Simulation #2: decline and fall
This nation enjoys a slight initial spurt of growth in output per capita, but after fewer then 20 years per capita income enters into a perpetual decline. The problem arises because the slow rate of technological advance is coupled with a high rate of population growth.
Parameter values: $\lambda = 0.65$, $\lambda' = 0.2$, $s = 5.25\%$, $n = 5\%$, $\rho = 0.1\%$,
Equilibrium values: $q_e = k_e = 4.2\%$; $(Q/K)^e = 0.8$; $q - n = -0.8\%$.

Table 12.4. Growth model simulation #2.

Parameter values: $\lambda = 0.65$, $\lambda' = 0.2$, $s = 5.25\%$, $n = 5\%$, $\rho = 0.1\%$, $K(0) = 20$, $N(0) = 100$, $R = 10$							
Year	N	K	Q	Q/K	q	k	Q/N
0	100	20	51	2.57			0.51
1	105	23	54	2.40	6.0%	13.5%	0.52
5	128	35	68	1.93	5.4%	10.6%	0.53
10	163	55	87	1.59	5.0%	8.6%	0.53
25	339	152	174	1.15	4.6%	6.1%	0.52
50	1,147	557	512	0.92	4.3%	4.9%	0.45
75	3,883	1,719	1,454	0.85	4.2%	4.5%	0.37
100	13,150	4,992	4,077	0.82	4.2%	4.3%	0.31
150	150,798	39,629	31,672	0.80	4.2%	4.2%	0.21

$\lambda + \lambda' = 0.85 < 1$. A slowing of the rate of population growth, as might be achieved through emigration or the encouragement of population control, might arrest the decline in living standards. Or more rapid technological advance might be achieved by borrowing state of the art techniques from more advanced nations or encouraged with government subsidies or tax breaks for research. If nothing is done, the grim predictions of Malthus will prove all too true, the decline in living standards continuing until the wage is driven below the subsistence level and the rate of population growth, g, is checked by starvation or disease.

It is intriguing to note from equation (22) that the equilibrium growth rate does *not* depend on s, which is the proportion of output that is saved for investment rather than consumed. However, s does affect the equilibrium capital/output ratio and the level of consumption at any particular point of time. Since $Q_t^{(1-\lambda')} = Q_t/Q_t^{\lambda'} = \alpha(1+\rho)^t L_t^{\lambda}(K_t/Q_t)^{\lambda'} R^{1-\lambda-\lambda'}$ from (12),

$$Q_t = (\alpha R^{1-\lambda-\lambda'})^{\gamma}(1+\rho)^{\gamma t} L_t^{\gamma\lambda}(K_t/Q_t)^{\gamma\lambda'}\,, \tag{25}$$

where $\gamma = 1/(1-\lambda')$. Substituting from (12) and (24) yields

$$Q_t^e = (\alpha R^{1-\lambda-\lambda'})^{\gamma}(1+\rho)^{\gamma t} L_o^{\gamma\lambda}(1+n)^{t\gamma\lambda}(s/q^e)^{\lambda\gamma'}\,. \tag{26}$$

Also, since $C_t = (1-s)Q_t$,

$$C_t^e = (1-s)Q_t^e\,. \tag{27}$$

Thus the height of the full employment growth path and the equilibrium consumption path are both affected by the savings ratio. In the longer run, other things being equal, two nations that are similar in terms of the pace of technological advance and the rate of population growth but with different saving ratios will end up growing at the same rate. But the saving ratio does matter, because one country may always enjoy a higher standard of living than the other at every point of time. The saving rate can be too high as well as too low. It can be shown that a country with a savings rate $s > \lambda'$ could enjoy a higher consumption path by reducing its savings rate.[9] A country can conceivably save too much, but $s > \lambda'$ means that saving is larger than capital's share in the nation's output!

12.5.4 *Why not growth?*

Because differences in living standards among nations are so huge, understanding why some nations prosper while others stagnate is one of the most pressing economic issues of our time. Many argue that secure property rights are a precondition for convergence of living standards among nations — who will invest if private property is not protected? Rapid development is said to be more likely when a country opens its doors to international trade, to foreign investment, and to the adoption of new technologies. Our growth model suggested that lagging nations will find it easier to catch up if more advanced nations are willing to share their advanced technology with less developed nations.

How willing countries are to import new technologies from more advanced nations may go a long way toward explaining why some nations experience development miracles while others stagnate. Indeed, Stephen L. Parente and Edward C. Prescott argue that countries stagnate because their governments discourage the adoption of new technologies. They explain that constraints on the adoption of modern techniques arise from the monopoly rights possessed by "industry insiders with vested interests tied to current production processes."[10] That is to say, the monopolists resist the adoption of new technologies that will undermine their monopoly position. According to their theory, Britain was the first to industrialize because the shift in power away from the crown to Parliament

[9]Edmund S. Phelps, "The Golden Rule of Accumulation: A Fable for Growthmen," *American Economic Review*, September, 1961.

[10]Stephen L. Parente and Edward C. Prescott, *Barriers to Riches*, MIT Press, 2000, p. 131.

led to a decline in regulation and meant that no group could successfully block the adoption of improved technologies. France, in contrast, was not hospitable to industrialization because the crown had sanctioned monopolies that were protected by elaborate regulations. Parente and Prescott argue that Japan's development miracle, a catching-up increase in the 1950s and 60s in their standard of living from only 20% to 75% of the U.S. standards, occurred because after World War II the U.S. occupying forces broke up much of Japan's industrial bureaucratic complex and succeeded in creating a more competitive environment.

Columbia University Professor Jeffrey Sachs argues that geography is a major determinant of economic growth and welfare.[11] Tropical countries tend to be underdeveloped, with the notable exceptions of Hong Kong and Singapore. Agricultural production is 50% less efficient in tropical countries, in part because of pests and parasites, soil erosion and problems of water availability. And the tropical countries have been falling further and further behind. Around 1820 GDP per capita in tropical regions was about 70% of GDP per capita in temperate zones, but by the 1990s GDP per capita in the tropics was only 25% of GDP per capita in temperate zone countries.

12.5.5 *Real business cycles*

The simple neoclassical growth model we have been discussing generates a smooth and steady path of economic development. This unrealistic result arises from the simplifying assumption that technological progress is a smooth and unbroken path and that the economy is not perturbed from its growth path by wars and other disturbances. More than a half century ago Harvard Professor Joseph Schumpeter [1883–1950] had argued that cyclical departures from the long run equilibrium growth path to which the economy naturally converges were an inherent part of the process of economic evolution. According to Schumpeter, the business cycle was part of the natural process by which the inherently stable economy responds to the shock of technological innovation. Downturns and recession are part of a process of "*creative destruction*" which contributes to economic development by weeding out the weak and unfit business enterprises and insuring the survival of the fittest.

[11] Jeffrey Sachs, "Tropical Underdevelopment," NBER Working Paper No 8119, February, 2001.

In 1982 Finn Kydland of Carnegie-Mellon University and Edward C. Prescott of the University of Minnesota published a pioneering article that led to the establishment of the Real Business Cycle school of macroeconomics.[12] Their analysis generated conclusions similar in a number of respects to Schumpeter's, but their analysis was based on a very sophisticated analytical foundation. They carefully developed the microfoundations of their model, invoking the Lucas supply function and rational expectations. They assumed that wages and prices are so flexible that they adjust promptly to balance the supply and demand for labor, arguing that fluctuations in the employment over the business cycle reflect voluntary adjustments in the labor supply to changes in real wages. Changes in the money supply, far from causing cyclical fluctuations, are endogenously generated when fluctuations in the pace of economic activity affect the demand for bank loans. According to the real business cycle theory, the economy is in continuous equilibrium, but equilibrium output fluctuates as a result of supply-side productivity shocks resulting from technological innovations and other disturbances, such as OPEC oil shocks and the aftermath of the September 11, 2001 terrorist attack. Arguing that the business cycle is the natural and efficient response of the economy to technological progress, real business cycle theorists conclude that attempts to smooth out the cycle, even if they worked, would be a mistake because they would generate harmful inefficiencies. Real Business Cycle theorists believe that recessions and unemployment are the price that must be paid for progress.

12.6 Population trends

The simplified neo-classical growth model presented in this chapter was optimistic about the future, provided that technological progress continues unabated. But admittedly, the model also involves a host of other major simplifications. In particular, it was assumed that the population grows at a constant rate, which is obviously far from the truth. Further, the model assumes that resources are never depleted! In fact, of course, some resources, such as oil, are subject to depletion while others, such as forests, are renewable. These issues deserve our attention. Let us start by looking at the population side of the Malthusian equation. What in fact has been the effect of rising worker productivity and higher living standards on the rate of population growth?

[12] "Time to build and aggregate fluctuations," *Econometrica*, 1982.

12.6.1 *The demographic transition*

Demographers analyze populations, gathering data, constructing models and interpreting population changes. Their studies of the *demographic transition*, the changes in the reproductive behavior of a country's population during the transformation from a traditional to a highly modernized state, reveal some surprising results. The demographers report that the classical assumption that the rate of population growth is an increasing function of the real wage could not be further from the truth. Quite the contrary, the transformation from a traditional pre-industrial society to a highly modernized state involves a dramatic rise in living standards coupled with a decisive *decline* in the birth rate. In the pre-industrialized state, high birthrates were balanced by low life expectancy. During the transition, mortality usually declined in advance of the decline in fertility, leading to a temporary spurt in the rate of population growth. It is generally true in most developed countries that women on average now bear only about half as many children as did their ancestors a century or two ago. But two centuries ago life expectancy was less than half of what it is today.

The decline in mortality in Europe, which began in the mid 18th century during the first phase of the demographic transition, resulted in large measure from dramatic improvements in health care, including improved sanitation, the pasteurization of milk, and vaccination for smallpox as well as tremendous advances in medical science. But what caused the decline in the birth rate? The customary explanation is that in earlier times children were a resource. They began work at an earlier age and were soon contributing more to the family than they consumed. Having a large family was also a means of providing for one's support if one is so fortunate as to live into old age. Contrast this with an advanced economy where child labor is generally outlawed, the costs of educating one's children can be substantial, and the emancipated young are said to make many trying demands on their parents. Further, lower child mortality rates reduce the number of births needed to achieve a family of targeted size. In addition, the development of pensions and social security provides an alternative to children as a source of support in one's old age.

According to projections by the United Nations Population Division, the population of the world is likely to grow from 5.7 billion in 1995 to about 9.4 billion in 2050 and 10.4 billion in 2100. The share of the world's population living in the currently more developed regions will decrease from

Table 12.5. Old age dependency ratios.

Year	Retirement age (65 and older) as a percentage of the working age population (20–64 years)						
	Canada	France	Germany	Italy	United Kingdom	United States	Japan
2000	20%	28%	25%	28%	27%	21%	27%
2025	36%	41%	36%	43%	36%	33%	37%

Source: United Nations.

19% to 10% in the next half century.[13] And declining fertility and mortality rates will lead to dramatic changes in the age-structure of the population. The share of the world population aged 60 and above will increase from 10% to 31% between 1995 and the middle of the 21st century. Table 12.5 shows how the old age *dependency ratio* is expected to increase in the next quarter century. For example, in Canada today there are about five people of working age for every senior citizen, but by 2025 there will be fewer than three working people for every senior citizen. No wonder many countries are worried about the financial viability of their social security systems. There will be major changes in career opportunities. One can expect to see a decline in the demand for teachers and an increase in demand for geriatric medical specialists and morticians.

12.6.2 *A simple overlapping generation model*

It is obvious that improved longevity will increase the average age of the population and may stress the financial viability of retirement programs. It is not so obvious that a reduction in the birth rate may have similar consequences. The hypothetical data presented on Table 12.6 are far from realistic, but they suffice for showing how changes in the rate of population growth can profoundly affect the age composition of the population, cause major shifts in the job market, influence the supply of aggregate savings and threaten the solvency of social security programs.

[13]The Population Division of the Department of Economics and Social Affairs at the United Nations Secretariat prepares population estimates and projections. Because long range projections are quite sensitive to changes in fertility rates, the demographers prepare low, medium and high estimates of likely population growth for alternative assumptions about fertility. Only the medium-fertility estimates are reported here. For more information, see
http://www.undp.org/popin/wdtrends/wdtrends.htm#World Population Estimates & Projections.

Table 12.6. Simple dynamics of population growth.

Panel 1: Population characteristics of Never-Never Land

Census year	Age bracket 0–20 youth	20–40 working	40–60 years	60–80 Retired	Total population	Adult (20+)	Average age	Retired/ working
1900	600	300	150	75	1,125	525	24.7	16.7%
1920	1,200	600	300	150	2,250	1,050	24.7	16.7%
1940	2,400	1,200	600	300	4,500	2,100	24.7	16.7%
1960	4,800	2,400	1,200	600	9,000	4,200	24.7	16.7%
1980	4,800	4,800	2,400	1,200	13,200	8,400	30.0	16.7%
2000	4,800	4,800	4,800	2,400	16,800	12,000	35.7	25.0%
2020	4,800	4,800	4,800	4,800	19,200	14,400	40.0	50.0%
2040	4,800	4,800	4,800	4,800	19,200	14,400	40.0	50.0%

Assumptions:

Everyone lives to be 80.

Until 1960 ever young working couple has four children.

After 1960 every young couple has two children (The pill or Roe vs Wade ???)

Panel 2: The teacher market in Never-Never Land

Census year	Youth	Students	Teachers	Teacher age 20–40	40–60	Average	Teacher/ Worker	Student/ Population
1900	600	300	15	10	5	36.7	3.3%	26.7%
1920	1,200	600	30	20	10	36.7	3.3%	26.7%
1940	2,400	1,200	60	40	20	36.7	3.3%	26.7%
1960	4,800	2,400	120	80	40	36.7	3.3%	26.7%
1980	4,800	2,400	120	40	80	43.3	1.7%	18.2%
2000	4,800	2,400	120	80	40	36.7	1.3%	14.3%
2020	4,800	2,400	120	40	80	43.3	1.3%	12.5%
2040	4,800	2,400	120	80	40	36.7	1.3%	12.5%

Assumptions:

50% of those in the 0–20 age bracket are students.

The student/teacher ratio is 20 to 1.

Panel 3: Consumption and saving in Never-Never Land

Census year	Adults	Workers	Consumption Income	Spending	Saving $S = Y - C$	Saving Ratio (S/Y)	Proportion Pop Retired
1900	525	450	1,350	1,050	300	22.2%	6.7%
1920	1,050	900	2,700	2,100	600	22.2%	6.7%
1940	2,100	1,800	5,400	4,200	1,200	22.2%	6.7%
1960	4,200	3,600	10,800	8,400	2,400	22.2%	6.7%
1980	8,400	7,200	21,600	16,800	4,800	22.2%	9.1%
2000	12,000	9,600	28,800	24,000	4,800	16.7%	14.3%
2020	14,400	9,600	28,800	28,800	0	0.0%	25.0%
2040	14,400	9,600	28,800	28,800	0	0.0%	25.0%

Assumptions:

Annual Wage Rate = \$3.00.

Each citizen's Lifetime Consumption = Lifetime Income (i.e., C = \$2.00 for both workers and retirees).

Panel 1 of Table 12.6 presents population data for Econoland, a mythical country where a census is taken every twenty years. In this grossly simplified economy, every family has four children and everyone lives for exactly 80 years. Everyone enters the work force at age 20 and works until age 60. Observe from the first several censuses that the population had been doubling every 20 years, thanks to the decision of every couple to have four children. Although everyone lives until age 80, the average age is not 40. The average age of the population is only 24.7 years because each successive generation is twice as large as its parents' generation.

Population momentum

After the 1960 census, perhaps as the result of the invention of a pill or the legalization of abortion, the number of children per family drops to two. As can be seen from the table, the number of citizens in the 0–20 age bracket stabilizes. But until 2020, the population continues to grow. *Zero population growth (ZPG)* is not reached until year 2020, when the cohort consisting of the 2,400 children born just in time to be measured in the 1940 census has died off and been replaced by a new cohort of 4,800 children. Thus the table illustrates the concept of *population momentum*: a long transitional period must pass before a change in child bearing behavior or mortality has its full impact on the rate of population growth and the age composition of the population. Adapting to the consequences of a change in the birthrate or mortality can require several generations.

Panel 1 also reveals several surprising demographic shifts. The average age increases from 24 to 40 as a result of the shift to zero population growth. And the ratio of retirees to workers rises from 1/8 to 1/2, which threatens the financial viability of the social security system. But while a higher percentage of the population is in retirement, the child proportion has shrunk and so the dependency ratio (the number of children plus retirees divided by the working population) is far below the rate prevailing in earlier times when the population was growing so rapidly.

Teacher job market

Panel 2 of Table 12.6 examines how the reduction in family size affects the job market for teachers. Under the assumption that half in the 0 to 20 age bracket are in school and that the average student teacher ratio is 20 to 1, 3.3% of the working population was in the teaching profession, with an average age of 36 years when the typical family had four children.

The population slowdown has a dramatic effect on the demand for new teachers. The 1980 census reports that there are only 40 teachers in the 20–40 age bracket — college graduates in the 80s or 90s who wanted to enter the teaching profession found that very few teachers were being hired — some would-be teachers joined the growing elder-care professions instead. Because the younger generation was unable to enter the teaching profession, the average teacher age climbed to 43 years. Once Zero Population Growth is reached, the proportion of workers who are employed as teachers is only 1.3%, less than half of the 3.3% in the days of larger families and steady population growth.

Saving ratio

Panel 3 of our table investigates the effect of zero population growth on the saving ratio. It is assumed that workers provide for their retirement by saving one-third of their income, which allows them to consume the same amount in retirement as when they were working.[14] With rapid population growth, the economy's savings rate was a high 22.2% because a very small proportion of the population was dissaving in retirement. With ZPG, 1/3rd of the adults are dissaving in retirement and the aggregate savings ratio is zero! This does not necessarily mean that the country will suffer from under-saving or over-consumption. A mature economy needs much less savings because it does not have to put as much aside for investment in the tools and equipment that were needed in the past for the growing generations of new workers. Thus the decline in the savings ratio may not be a bad thing!

12.7 Exhaustible resources

How soon will we run out of oil? Will the market mechanism appropriately allocate non-renewable resources (e.g., oil) over time? Many environmentalists answer such questions with a resounding no, arguing that government intervention is required to prevent the exploitation of our finite resources and protect the interests of future generations. The Solow style growth model we analyzed is not capable of analyzing these complications because

[14]In the Modigliani-Brumberg life cycle model of consumption, briefly discussed in Chapter 9.5.1, it is assumed that consumers put aside enough each year to be able to enjoy the same standard of living in retirement as in their working years but have no bequest motive.

it did not allow for exhaustible resources, such as coal and petroleum. A model appropriate for analyzing how the market allocates non-renewable resources, pioneered by Harold Hotelling, provides a surprisingly optimistic answer to these questions.[15]

12.7.1 *Two numerical examples*

We shall use a simple numerical example in analyzing exhaustible resources. Suppose there are 100,000 barrels of oil in Never-Never Land. The initial price of oil is $p_0 = \$5.00$ per barrel at the well-head and the rate of interest (borrowing or lending) is $i = 7\%$. How much oil should we pump next year? How much should we save for future generations? What will the market decide?

Suppose you own an oil well in Never-Never Land. Should you pump all your oil today for \$5.00 per barrel or should you leave it in the ground? The answer depends on what you think will happen to the price of oil. If you expect the price of oil to rise to less then \$5.35 in year two, you should pump all your oil today, sell it for \$5.00 per barrel and place the money in the bank earning 7% interest so as to get back \$5.35 in year two for each gallon of initial wealth. And other producers, if they have the same expectations you do, will also pump today, which will tend to push down today's price and raise tomorrow's. If, on the other hand, the price is expected to rise to more than \$5.35, you should leave your oil in the ground and pump tomorrow rather than hold money in the bank at only 7%. Further, if a speculator anticipates that the price of oil will rise by more than 7%, she can borrow from the bank at 7%, buy some oil, and inventory it for a year in order to turn a neat profit. All this means that competition among oil producers and speculators will tend to push next year's price to 1.07 times this year's price.

This argument underlies Professor Hotelling's proposition that in a competitive market with accurate expectations the price p_t of oil (net of extraction and refining costs) will increase at the same rate as the rate of interest:

$$p_{t+1} = (1 + i)p_t \,. \tag{28}$$

[15] Harold Hotelling "Economics of Exhaustible Resources," *Journal of Political Economy*, April 1931.

The price of gas at the pump might rise at a faster or lower rate, depending on the cost of extraction, transportation, and distribution, but the core price of petroleum rises at the same rate as money in the bank.[16]

It will be constructive to consider a numerical example. The initial stock of oil is 100,000 barrels and the demand function is $q = 50000p^{-1.1}$. Table 12.7 shows what happens. Whoops! The stock of oil was exhausted in year 25.

What went wrong? The problem is that the initial price of \$5 per barrel was too low. As a result there was excessive consumption in year 0, and since the price was rising at 7% per annum in accordance with equation (28), oil was under priced in future years as well! One suspects that long before the stock of oil was exhausted the rising consumption/stock ratio would signal that something was wrong. Speculators will sense an opportunity for profit. Rational speculators, anticipating the future shortage, will purchase

Table 12.7. Excessive depletion in Never-Never Land.

Initial oil stock = 100,000, Interest rate: 7.00%; Demand = $50000p^{-1.1}$				
Year	Price	Demand	Stock	Demand/Stock
0			100,000.00	
1	\$5.00	8,513.40	91,486.60	8.5%
2	\$5.35	7,902.80	83,583.80	8.6%
3	\$5.72	7,335.99	76,247.81	8.8%
4	\$6.13	6,809.83	69,437.98	8.9%
5	\$6.55	6,321.42	63,116.56	9.1%
6	\$7.01	5,868.03	57,248.54	9.3%
7	\$7.50	5,447.16	51,801.38	9.5%
8	\$8.03	5,056.48	46,744.90	9.8%
9	\$8.59	4,693.81	42,051.09	10.0%
10	\$9.19	4,357.16	37,693.93	10.4%
11	\$9.84	4,044.65	33,649.27	10.7%
12	\$10.52	3,754.56	29,894.71	11.2%
13	\$11.26	3,485.28	26,409.43	11.7%
14	\$12.05	3,235.30	23,174.13	12.3%
15	\$12.89	3,003.26	20,170.87	13.0%
23	\$22.15	1,655.83	2,731.58	37.7%
24	\$23.70	1,537.07	1,194.51	56.3%
25	\$25.36	1,426.83	0	119.4%

[16]The real rate of interest should be used if the price of oil is properly deflated by a general price index.

more oil and store it in anticipation of a tidy profit when the price rises more rapidly in response to the growing shortage. The higher price hike generated by spectator purchases will hold current consumption in check, thereby leaving more for future generations.

Table 12.8. Market equilibrium insures oil forever more.

Initial oil stock = 100,000; Interest rate: 7.00%; Demand = $50000p^{-1.1}$				
Year	Price	Demand	Stock	Demand/Stock
0			100,000.00	
1	\$5.84	7,172.25	92,827.75	7.2%
2	\$6.25	6,657.83	86,169.92	7.2%
3	\$6.69	6,180.32	79,989.60	7.2%
4	\$7.16	5,737.05	74,252.55	7.2%
...				
19	\$19.75	1,878.70	24,315.27	7.2%
20	\$21.13	1,743.95	22,571.32	7.2%
Sum to Infinity: 100,000.00.				

The next example is similar to that on Table 12.7, except that the initial price is \$5.84. At this price consumers will demand 7,172 barrels of oil in year 1. Thus the higher initial price is leading to more oil being put aside for future generations. And as is clear from the table, from this start 7.2% of the stock is consumed each year forever more. We never run out of oil! True, oil gets more and more expensive, and as its price rises consumers purchase less and less, perhaps by making use of alternative renewable resources, such as firewood, or hydroelectric power.

12.7.2 *Analysis*

Now we must examine how the size of the initial oil stock, the elasticity of demand for this exhaustible resource, and the rate of interest interact to determine the current price of petroleum. The elasticity of demand turns out to be critical, because it indicates how willing the public is to cut back on oil consumption when its price rises. The demand elasticity together with the rate of interest will determine how rapidly the public is weaned from petroleum.

Suppose that the demand for petroleum in year t is

$$q_t = \alpha p_t^{\eta}\,, \quad \eta < 0 \tag{29}$$

and that the price changes in accordance with Hotelling's principle at rate $i > 0$; i.e.,

$$p_t = (1+i)p_{t-1} = (1+i)^t p_0 \,. \tag{30}$$

Substituting into the demand equation reveals that the quantity of oil demanded in period t is

$$q_t = \alpha p_t^\eta = \alpha(1+i)^{\eta t} p_0^\eta \,. \tag{31}$$

To simplify notation, we let $\beta = (1+i)^\eta$ so that $p_t = \beta p_{t-1} = \beta^t p_0$ and equation (31) becomes

$$q_t = q_0 \beta^t \,. \tag{32}$$

If the initial stock of oil Q_0 is to be consumed over the infinite future with prices rising at rate i, we must have

$$Q_0 = q_0 + q_1 + q_2 + \cdots + q_t + \cdots = q_0(1 + \beta + \beta^2 + \ldots) \tag{33}$$

Now the expression in parentheses is the sum of a geometric series which must converge because $0 < \beta < 1$; therefore,

$$Q_0 = \frac{q_0}{(1-\beta)} \,. \tag{34}$$

Therefore, we must have first period consumption of

$$q_0 = (1-\beta)Q_0 = [1 - (1+i)^\eta]Q_0 \,, \tag{35}$$

which is achieved, given the demand function (31), with a first year price of

$$p_0 = \left(\frac{q_0}{\alpha}\right)^{1/\eta} = \left\{[1-(1+i)^\eta]\frac{Q_0}{\alpha}\right\}^{1/\eta} \tag{36}$$

Two points about this expression deserve special attention:

1. The smaller the price elasticity of demand, the lower the initial consumption, the higher the initial price and the less rapidly consumption is cut back with the passage of time, given the rate of interest.
   That is to say, the more vital the exhaustible resource is for our consumption, perhaps because there are few close substitutes, the less we should consume today and the more we ought to put aside for future generations.

2. The lower the interest rate the less we should consume today because the price will rise more slowly and the drop off in consumption will be less rapid, other things being equal.

This also makes sense. We provide for future generations in part by limiting our consumption of non-renewable resources and in part by accumulating physical capital, such as factories and equipment and housing. The interest rate, because it reflects the marginal productivity of funds invested in new capital equipment, balances these alternative means of providing for the future. If the marginal productivity of capital is high, investment in more productive capital equipment will provide for future generations more effectively than a larger petroleum stock. If the interest rate is low, investment in physical capital is less effective than providing for future generations by heightened conservation of non-renewable resources.

12.7.3 *Moral*

The moral of Hotelling's model is that competitive markets work to allocate non-renewable resources appropriately over time, giving proper weight to the interests of future generations. But the model is subject to serious question because it does not correctly predict the movements in the price of oil in recent decades. The price of petroleum products has increased at much too low a rate. Indeed, Table 8.8 revealed that the real price of gasoline in the United States was lower in 2000 than it had been in 1960. Hotelling's elegant principle is blatantly inconsistent with the facts. How can this be?

Part of the discrepancy may arise because the Hotelling principle applies only to the price of the raw material itself, while the price of gasoline at the pump reflects as well the costs of exploration, extraction, refining and distribution, which are not subject to the Hotelling principle. This means that the retail price of petroleum products should increase less rapidly than the rate of interest. Furthermore, rapid technological advance in exploration and drilling procedures have substantially reduced the cost of extraction.

While these two factors help to explain why petroleum prices have increased less rapidly than a naïve application of the Hotelling model would suggest, they are not the full story. Many of the richest sources of oil today are in countries that are politically unstable. Or in the jargon of economists, property rights are insecure. When revolution threatens, prudent oil barons will extract petroleum more rapidly because it is much safer to have your

funds deposited in a Swiss bank account than to hold oil underground in the hope that you and your children will be able to get a better price in the future. The proper conclusion to be reached from Hotelling's model is that the market mechanism will work to allocate the use of non-renewable resources appropriately over time, but only if property rights are secure.

12.8 Renewable resources — Over-fishing

While some resources are exhaustible, so what is consumed today is not available for future generations to enjoy, others are renewable. Properly cared for, farmland, the forests and the oceans may be harvested in perpetuity. For such resources, we shall find, competition is not always for the best.

Generations of fisherman have assumed that the treasures of the sea are inexhaustible, but today it is all too clear that fishing resources are finite. Thanks in part to improved technology — electronic equipment for locating schools of fish, larger boats, and better nets — over-fishing has become a problem, and many fish stocks are collapsing. Here are some of the consequences:[17]

- In the past 20 years the average size of swordfish caught in the North Atlantic has dropped from about 265 pounds to 90 pounds. Most of the swordfish are being caught before they have had a chance to breed.
- With the passage in 1977 of the Magnuson Act, the U.S. took control of marine resources within 200 miles of the coast, driving out the factory ships from the Soviet Union, Japan, and Spain. But by 1980 the New England fishing fleet had expanded by 42% to take up the slack.
- After the Grand Banks stock of cod collapsed in the early 1990s, Canada closed its centuries old cod fishery, putting 30,000 people in Newfoundland out of work.
- The New England Groundfish Recovery Plan allows fishermen to spend only 88 days at sea each year catching groundfish.
- Georges Bank, 100 miles southeast of Cape Cod and for 500 years the worlds richest fishing ground, was closed to ground fishing when its stock of cod and haddock collapsed in the early 1990s, costing New England $350 million a year and 14,000 jobs.

[17] *Hartford Courant*, 11/27/98

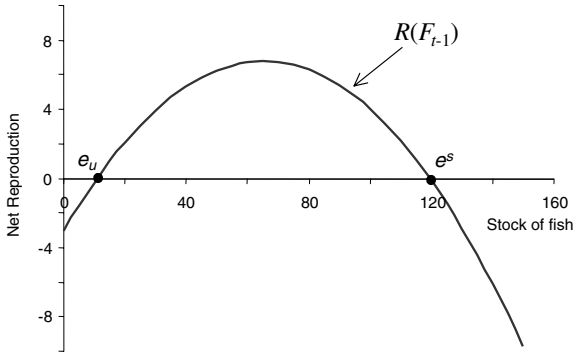

Fig. 12.8. Fish population and the balance of nature
The net-reproduction function $R(Ft-1)$ shows how the change in the stock of fish from one year to the next (the excess of births over deaths) depends on the number of fish already in the lake.

Point e_s is a stable equilibrium point characterized by zero population growth (ZPG). This point represents the balance of nature.

Point e_u is an unstable equilibrium. If the stock falls below e_u, the net-reproduction rate will be negative and extinction threatens.

Clearly, when the bounty of the sea is abused, the decline in the stock of fish is disruptive. It involves not only the loss of an important source of protein. It can also put thousands out of work. Fisherman must go further out to sea to make their catch. They now face the perils of collapsing fishing grounds as well as the traditional dangers of the sea. Restrictions on the number of days of fishing lead many fisherman to take on second jobs in the construction trades or elsewhere, but the boats are idle.

12.8.1 *Balance of nature*

In order to analyze some essential aspects of the problems of renewable resources, we shall construct a simple graphical model explaining the size of the population of a single species of fish.[18] The number of fish in the sea is plotted on the abscissa of Figure 12.8 while the change in the fish stock is on the ordinate. The curve labeled $R(F_{t-1})$ shows the net reproduction rate for the fish population. That is to say, $R(F_{t-1})$ is the excess of the

[18]For a clear mathematical analysis of renewable resources see David Levhari, Ron Michener and Leonard J. Mirman, "Dynamic Programming Models of Fishing: Competition," *American Economic Review*, September, 1981.

number of births over the number of deaths; it is the change in the number of fish from last year:

$$\Delta F_t = F_t - F_{t-1} = R(F_{t-1})\,. \tag{37}$$

If the population of fish is zero, it obviously remains at zero. Indeed a critical mass of fish (at least two) is required for reproduction to take place. Thus we must surely have $R(0) \leqslant 0$ and $R(1) \leqslant 0$. With more fish in the sea, more baby fish will be born and survive, and the faster the stock of fish will grow, but only up to a point. Once the fish population becomes large relative to the size of the sea, crowding and competition for food will reduce the number of young fish that survive and thrive. Therefore, for sufficiently large F_{t-1}, $d\Delta F_t(F_{t-1})/dF_{t-1} < 0$. More than this, if the fish population becomes much larger, the competition for food may be so intense that the number of deaths of older fish will exceed the number of replacement fish that hatch during the year; i.e., for F_{t-1} sufficiently large, $\Delta F_t(F_{t-1}) < 0$.

Point e_s on the graph represents zero population growth. If the fish population reaches magnitude e_s it will stay at that level indefinitely. Thus, e_s is an equilibrium fish stock. And it is a stable equilibrium: If there are fewer than e_s fish in the pond, the stock will expand; if there are more, the stock of fish will decline. Point e_u is also an equilibrium point, but it is unstable. If the number of fish is $e_u + \varepsilon$, $\varepsilon > 0$, $F_t > F_{t-1}$, and the population will continue to increase and we move further and further away from e_u; or if $\varepsilon < 0$, the population shrinks to zero (extinction). Point e_s may also be said to represent the *balance of nature*." Before fishermen began to harvest fish from the sea, the population will stabilize at e_s.

12.8.2 *Fishing*

Suppose that fishermen harvest Q fish from the sea each year. Let us see how this will upset the balance of nature. If Q fish are harvested each year, the fundamental equation describing how the population of fish changes is now

$$\Delta F_t = R(F_{t-1}) - Q\,. \tag{38}$$

What happens is shown on Figure 12.9. The ΔF_t function shifts down by the size of the annual catch. As a result, e_s is no longer viable. The population of fish will shrink toward the new equilibrium point e_s^*, and the fishermen can harvest Q fish from the sea every year forever more.

Over-fishing results if the fishermen try to harvest too many fish from the sea, as illustrated on Figure 12.10. The catch of magnitude Q is not sustainable, and the population of fish will "crash," threatening extinction! $Q^s = \max[R(F)]$ is the *maximum sustainable catch* that can be harvested from the sea for evermore.

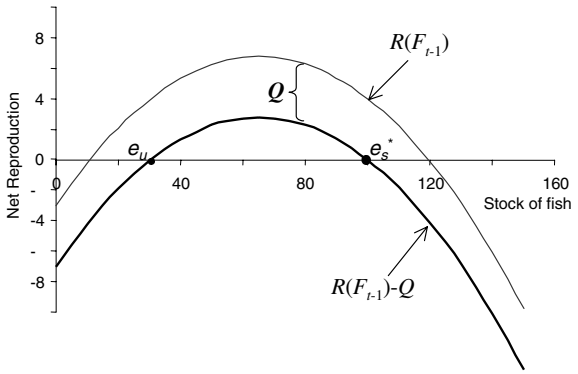

Fig. 12.9. Fishing
If fisherman harvest a catch of Q fish from the lake each year, the change in the number of fish from one year to the next will be $R(F_{t-1}) - Q$. As a result the equilibrium stock of fish is smaller than it was in the state of nature.

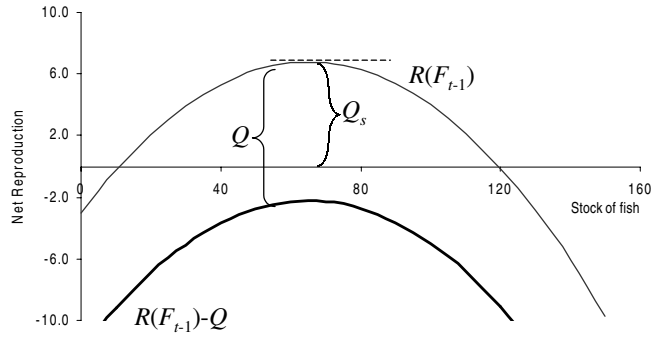

Fig. 12.10. Over fishing
Q_s is the maximum sustainable catch. When the catch Q exceeds the maximum sustainable catch, there is no equilibrium and the stock of fish shrinks. The fish stock may crash. Extinction threatens.

12.8.3 *Market equilibrium*

Assuming that fishing is a competitive industry with free entry, whether over-fishing occurs or not depends in part on the demand for fish and in part on the costs involved in harvesting the sea. Let $D(p)$ denote the demand function for fish and $S(p, F)$ the supply function. The stock of fish is included in the supply function because the more fish there are the easier they are to catch; which means that $\partial S/\partial F > 0$. On Figure 12.11 we have the demand curve and some representative supply curves. The more fish in the sea the larger the catch and the lower the price, as indicated by the succession of market equilibrium points on that graph. The short-run equilibrium price $p(F)$ and catch $Q(F)$ are determined by the market clearing requirement that $S(p, F) = D(p)$, This dependency of the size of the catch on the stock of fish, $Q(F)$, is plotted on Figure 12.12. The resulting long-run equilibrium stock of fish and the corresponding sustainable market determined catch is indicated by point e where $Q(F) = R(F_{t-1})$.

While the market determined catch displayed on Figure 12.12 is sustainable, this is not necessarily the case. The $Q(F)$ function will shift upwards, signifying that at any given stock of fish more will be harvested, if the demand curve for fish shifts upwards because of rising incomes or an expanding population. The $Q(F)$ function will also move upwards if the development of more efficient fishing techniques causes the supply function to shift downwards The result may be a crash in the stock of fish

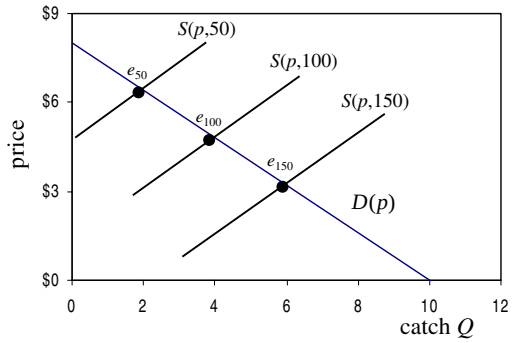

Fig. 12.11. Fish market equilibrium, short run
The more fish there are in the sea, the easier they are to catch. The family of short run supply curves, $S(p, F)$, reveal how the quantity of fish that will be brought to market as a function of the price depends on the number of fish in the sea. The short-run equilibrium points show how the price and quantity are simultaneously determined, given the stock of fish.

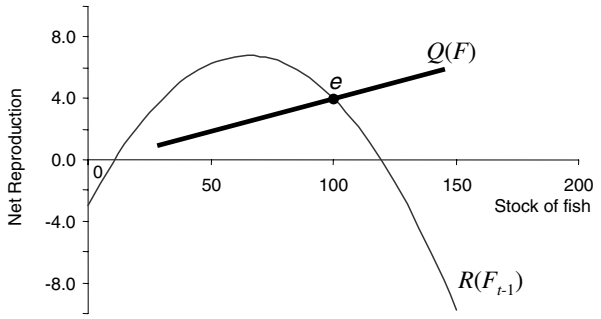

Fig. 12.12. Fish market equilibrium, long run
The $Q(F)$ function, derived from Figure 12.11, shows how the size of the catch depends on the number of fish in the sea. At long run equilibrium point e, the size of the catch just equals the reproduction rate; i.e., $Q(F^e) = R(F^e)$.

A large upward shift in the demand for fish, function $D(p)$ on Figure 12.11, could push the $Q(F)$ function above $R(Ft-1)$. There would no longer be a fishing equilibrium point. Competition would generate over-fishing and cause the fish stock to crash. A tax on fish, a tax on fishing boats, quotas restricting the size of the catch, or a shortening of the fishing season might save the fish from extinction.

and disaster for the fishing industry. Measures that reduce $S(p, F)$, such as a tax on fishing or government regulation, such as quotas restricting the size of the catch or shortening the fishing season, may be the only way to prevent extinction.

Competition does not work to obtain economic efficiency in the case of fishing. The market mechanism fails to take into account an important cost of fishing activity. The external diseconomy in fishing arises from the fact that my fishing activity, by reducing the stock of fish, leads to an increase in your fishing costs. While all fishermen would gain from more limited fishing activity, each profit maximizing fisherman takes only his own fishing costs into account. A conservation aware fisherman who restricts production in the interest of future generations will suffer a loss in profits, unless all the other fisherman also sacrifice immediate profit by cutting back on their fishing activity.

12.9 Conclusions

This chapter has looked at a number of models. Some were failures. The classical model erroneously predicted stagnation at the subsistence level because the force of technological change and the contribution of capital

accumulation were grossly underestimated. Hotelling's analysis of the way in which the market efficiently allocates petroleum and other resources in fixed supply erroneously predicted that the price of oil would increase overtime at a pace equal to the rate of interest. But even models that fail can be informative. The failure of the classical model teaches that the forces of technological change and capital accumulation must never be underestimated. The failure of Hotelling's model warns that when property rights are insecure the market cannot be relied upon to allocate our finite petroleum resources efficiently over time. The model used to analyze renewable resources — e.g., fish — did not fail. Its predictions are all too true. Once again we find that when property rights are insecure, competition can be extremely harmful. The neo-classical growth model provided a framework for analyzing what factors contribute to the growth of nations and the conditions necessary for the perpetual improvement in economic well-being.

Summary

1. Thomas Malthus argued that populations tend to grow geometrically while food production grows arithmetically, which means that the world will inevitably run out of food in the absence of population control. Adam Smith, David Ricardo and other members of the classical school worried that the economy would approach a stationary state characterized by a subsistence standard of living because the growing population would push the wage rate down to the subsistence level. The first growth model developed in this chapter approached the classical stationary state by assuming diminishing returns, by assuming that the rate of growth of population was proportional to the excess of the wage over the subsistence level, and by ignoring technological progress.
2. A neo-classical model of the growth process generated more optimistic results. This model assumed a constant rate of population growth and diminishing returns, but these factors could be offset by sufficiently rapid technological progress. It was shown that the model could approach an equilibrium characterized by a constant rate of output growth and a stable capital/output ratio. While the saving ratio does not affect the rate of growth of per capita income, it does affect the height of the equilibrium consumption growth path.
3. The neo-classical growth model predicts that poorer countries will tend to catch up with their wealthier neighbors if they have similar natural

resources, the same rate of population growth, and are able to adopt the technology enjoyed by their more advanced neighbors. But equation (22) revealed that decline rather than growth might be generated if the rate of technological progress is too low relative to the degree of diminishing returns. Parente and Prescott argue that countries fail to develop when vested interests protecting their investments in outdated technology prevent the adoption of more efficient production techniques. Jeffrey Sachs explains that tropical countries tend to be underdeveloped because agriculture is less efficient and disease is epidemic in tropical climates.

4. In contrast to classical theory, demographic studies reveal that populations grow less rapidly when wages move above the subsistence level. A simple overlapping generations model showed that it would take several generations for the population to stabilize after a change in the birth rate. A fall in the birth rate would lead to the aging of the population, subject the social security system to financial stress, adversely affect the market for teachers and cause the savings rate to decline.
5. The simple neo-classical growth model developed in this chapter did not allow for exhaustible resources, such as oil. The model of Hotelling predicts that the price of petroleum will rise at a rate equal to the rate of interest. As a result the consumption of oil will decline geometrically, but we will never run of out of oil. This has not happened to the price of oil, in part perhaps because the failure to secure property rights has encourage excessively rapid depletion of petroleum resources.
6. Fish are a renewable resource, but they are subjected to over-fishing, resulting in the collapse of fishing stocks in many areas of the world and substantial economic loss to fisherman. The problem of over-fishing is said to arise because fish are a common resource unprotected by private property rights, which means that the market mechanism cannot function to allocate this scarce natural resource appropriately.

Key Concepts

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Exercises

1. Suppose that output Q grows at rate $q = 4\%$, K grows at rate $k = 5\%$, and the population grows at rate 3%. How rapidly will Q/K change? How rapidly will per capita output grow?
   Hint: Consider footnote 3.
2. Never-Never Land has the following production function: $Q = (1.02)^t L^{2/3} K^{1/3}$.
   The savings rate is 10%. The labor force grows at 1% per annum.
   a. Derive the equilibrium growth rate of output, assuming that labor is always fully employed.
   b. Determine the equilibrium output/capital ratio.
   c. What is the equilibrium rate of growth of per capita income?
3. Congratulations, you have inherited an oil well from your late Uncle Rich. It is estimated to hold 500,000 barrels of oil. And the current price of oil is $20 per barrel. You could pump the oil out of the well and put the money in a Swiss bank account, where it would earn 10% interest. Or you could leave the oil in the ground and pump it next year or ever further in the future if you like.
   a. How much oil will you pump this year *if* you think that next year the price of oil will be 12% higher than it is today? Explain why.
   b. Suppose that oil well owners pump some but not all of their oil out of the ground this year — some oil is left in the ground for another year. Suppose also that all oil well owners have the same expectations about future price increases. What must be the expected rate of increase in the price of oil?
4. The following function describes the net reproduction rate for fish in Lost Lake
$$R(F_{t-1}) = [25 - (F_{t-1} - 5)^2]^{1/2} - 1\,.$$
   a. Evaluate the net reproductive rate for $F_{t-1} = 0$, 5, 8, 9 and 10.

b. Plot the net reproduction rate as a function of F_{t-1} on a neat graph.
Hint: $(R+1)^2 + (F_{t-1} - 5)^2 = 25$ is the equation for a circle.

c. Determine the number of fish in Lost Lake in the state of nature (stable equilibrium).

d. Some fishermen find Lost Lake. Determine the equilibrium stock of fish if they catch only three fish per year.

e. Determine the maximum sustainable catch.

f. What will happen if our greedy fishermen catch six fish every year?

5.#* Solve the classical model summarized by equations (2) and (4) by finding the function $L_g = f(g, L_0)$.
Hint: Note from equation (4) that $\ln w_g = \ln(\lambda\rho R^{1-\lambda}) + (\lambda - 1)\ln L_g$. Substitute this expression into the ln transform of equation (2) in order to obtain a first order linear difference equation.

6.# Show that a savings rate equal to λ' will yield the maximum sustainable equilibrium consumption path characterized by a constant rate of growth.
Hint: Substitute equation 26 into 27 and differentiate with respect to s.

7.#* For the very first exercise in this book you were asked to write a summary of an article in a professional economics journal that was of particular interest to you. Go back and reread the article and your critique. You may be surprised at how much more sense that article makes to you now that you have finished this book. If you did not do this project when you finished Chapter 1, go back and reread the assignment (question 1.1) and do it now.