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The Business Enterprise: Theory of the Firm

5.1 Introduction

In 1975 William H. Gates III and Paul G. Allen founded Microsoft as a partnership. That same year they shipped, as their first product, an

implementation of the BASIC language interpreter — this was the first computer language program to run on a personal computer. In 1981 they reorganized as a privately held corporation, Microsoft, Inc. In 1986 Microsoft went public with an initial public offering that raised $61 million. In May 1998, the Antitrust Division of the U.S. Department of Justice and a group of 20 state Attorneys General filed two antitrust cases against Microsoft. In 1999 Microsoft realized profits of $11.891 billion on sales of $19.747 billion, employing more than 34,000 people and incurring taxes of $4.106 billion. That, in a nutshell, is the twenty-five year history of what is arguably the most successful business enterprise in the history of capitalism.

This chapter starts with an explanation of the various forms of business organization, including a discussion of the differences between partnerships and corporations. You will also learn basic accounting principles that business enterprises use to report their financial condition to their owners and the tax authorities.

Later in this chapter we shall develop the theory of the firm. This basic engine of analysis examines the implications of the assumption that business firms maximize profits. We will ask how a profit maximizing firm will respond to an increase in wages or interest rates and how its pricing decisions may be influenced by a change in taxes.

The concluding section discusses Management Science, a discipline concerned with the application of mathematical techniques to the practical problems of business. This section considers the problem of efficient inventory management and explains the Just-in Time style of production management pioneered by Toyota.

Left for Chapter 6 is a discussion of alternative types of market organization and a report on the procedures used by governments to limit and control monopolies in an attempt to establish an environment for business in which profit maximization will best contribute to the benefit of society.

5.2   Organization of the firm

5.2.1   *Types of organizations*

Individual proprietorship

A student is recruited by one of her instructors to develop a web page. They agree that the student will receive $500 when the project is satisfactorily completed. With this arrangement, the student is not an employee. She is "self employed" — she has become a *sole proprietor*. This is the simplest

form of business structure. The instructor who recruits her to undertake the project will not have to worry about income tax withholding or social security taxes.[1] The student is legally required by the Internal Revenue Service to report her income on "*Schedule C: Profit or Loss from Business* (sole proprietorship)," which is a bit of a pain unless she uses *Turbo Tax* or some other income tax software to expedite the April 15th process. But using Schedule C may make it possible for her to deduct at least a part of the cost of her computer and the software used on the project when the time comes to calculate her profits from the project and the taxes that she owes.

Partnership

If our student's web page business prospers, she may find it advisable to recruit friends to help her with her projects. She might hire them as employees, she might have them work for her under contract (in which case her friends would also become sole proprietors[2]), or she might form a *partnership*. Any two or more people can agree to work together in a partnership. Physicians, lawyers, accountants and other professionals often adopt this form of organization.

When forming even the simplest of partnerships, it is essential that the participants draw up a written agreement carefully specifying the fraction of the work to be performed by each partner, the fraction of the capital that each shall provide, and how they will share profits, losses and debts. Even though the partners may have carefully specified their mutual obligations when they initially established the business relationship, it all too often happens that, after the initial enthusiasm for the business has worn thin, there will be a falling out, each partner thinking he is doing two thirds of the work and providing three quarters of the thinking.

A major limitation of the partnership mode of organization is that each partner assumes unlimited liability. Suppose your partnership fails. If you owned half the partnership, you will be assigned one half of the outstanding debts of the firm to be paid out of your own pocket. Worse than this, if

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[1] The professor may be required to file a Form 1099 with the IRS notifying them of the amount that the student is receiving. The student will have to pay the equivalent of the Social Security and Medicare taxes when she files her income tax return.

[2] The Internal Revenue Service has rather strict requirements that must be met before one can claim that one is a proprietor rather than an employee.

your partner lacks the financial resources to pay his half of the partnership's debt, you will be legally liable to shoulder the entire burden![3]

Corporations

Corporations are the dominant form of business organization. A corporation is an artificial "person."[4] At one time, anyone seeking to form a corporation had to obtain a charter through special legislative act. Thus, the British East India Company was established by a special act of Parliament in 1600. Today it is relatively easy in the United States to incorporate in any of the fifty states. Indeed, some law firms advertise on the back pages of the *New York Times* that you can incorporate over the phone for a fee of $99.95.

- One advantage of incorporation is that, unlike a partnership, the organization continues even if one of the investors dies — corporations attain immortality.
- Another advantage is that the owners of the corporation enjoy the right of *limited liability*. Limited liability means that if the corporation goes bankrupt, perhaps because the courts rule against it in a major suit, the investors only lose what they have already invested in the firm. While limited liability has a long history as a special privilege granted to those entrepreneurs enjoying political connections, the first general limited-liability law for manufacturing companies was passed by New York State in 1811. Other states quickly followed in order to avoid disadvantaging their own corporations, and Britain adopted limited liability in 1854. By reducing the risk of owning stock, establishing limited liability encouraged even passive investors to shift their funds from bonds to stocks. The

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[3]Young associate lawyers at major law firms aspire to make partner, but they are well advised to check out the liabilities of the firm and to be aware that they might be required to help pay any losses arising from pending lawsuits.

[4]The Fourteenth Amendment to the United States Constitution, ratified shortly after the Civil War, not only provided that all persons born in the United States, regardless of race, are citizens of the United States; it also provided that no state shall "deprive any person of life liberty or property without due process of law." The Courts subsequently interpreted the word "person" in the *due process clause* to include corporations and ruled that the clause prevented the states from regulating public utilities to the point where they receive anything less than a "fair rate of return." Economists serving as expert witnesses at public utility commission rate hearings are amply compensated for testifying about what constitutes a "fair rate of return."

establishment of limited liability made it much easier for corporations to raise the funds required for financing large-scale industrial capitalism.

- The *Corporate Profit Tax* is the primary disadvantage of incorporation. The federal tax is 15% on the first $50,000 of net income, but it rises in steps to a maximum rate of 35% on taxable net income over $10 million — such is the price for immorality and limited liability.

As can be seen from Figure 5.1, there are many more sole proprietary businesses than there are corporations or partnerships. But corporations are much larger on average, so if we look at the size of firms, as measured by their sales receipts, the corporations clearly dominate. Corporations today are the dominant form of economic organization in all modern industrial societies.

In addition to sole proprietorships, partnerships, and corporations, there are several other types of business organizations, some of which are more interesting than others. An *S Corporation* is a cross between a partnership and a corporation. If the participants in a business enterprise decide to form an S Corporation rather than a partnership they will receive the

Fig. 5.1.   Forms of business organization
The pie-chart on the left shows that sole proprietorships outnumber all other forms of business organization. The pie-chart on the right shows that the vast bulk of the nation's business is done by corporations.

Table 5.1.   Relative importance of different forms of business organization.

	Number (1,000)	Receipts (Billions $)	Receipts ($)/Firm
Proprietorship (nonfarm)	17,409	918	52,731
Partnership	1,855	1,534	826,954
Corporation	4,849	16,543	3,411,631

Source: Data for 1998 from *Statistical abstract of the United States*, 2001, Table 711.

benefit of limited liability, but they will not have to worry about the federal corporate profits tax. Instead, all the profits of the corporation will be "passed through" to count as current income of the shareholders. A major limitation of S Corporations is that there can be no more than 75 stockholders. In many states Limited Liability Companies (LLC) provide still another form of organization. In some states Chapter S corporations are subject to the state corporate profit tax but an LLC is exempt.

5.2.2   *How the modern corporation works*

The owners of a corporation are its shareholders, but what are shares? A corporation can raise funds by issuing shares of stock representing an ownership interest. For example, in year 2000 Microsoft had 5,250 million shares of stock outstanding. You could have purchased a share of Microsoft in 1990 for $21. In 1999 they had sold for as much as $119.93 per share, but after the District Court Judge ruled that Microsoft was guilty of serious violations of the antitrust laws, the stock declined dramatically in value to a low of $60.37.

You may have purchased shares in a corporation as an investment, but the act of purchasing also means that you are a part owner of the company; you have become a corporate *shareholder*. As part owner, you are entitled to vote for the corporation's *board of directors*. The Board in turn appoints the managers of the firm, oversees operations, and tries to insure that the company is run for the benefit of the stockholders. In corporate democracy it is not "one person one vote." The number of votes each shareholder may cast equals the number of shares that she owns. When the ballot arrives in the mail you may find that there is only one set of names on the ballot, those recommended by the incumbent directors. Critics complain that corporate democracy is like having only one political party — it is hard for dissatisfied stockholders to organize in opposition to the incumbent board of directors.

Microsoft is a publicly held corporation. Its shares are tradable on the NASDAQ stock exchange — that is, any individual with the money and the courage to invest may buy shares in the corporation. You can contact a stockbroker or purchase shares on the Net at the going market price. Small corporations are privately held. For example, if a construction firm partnership decides to incorporate in order to obtain the protection of limited liability, the owners may divide the shares up among themselves, but they will not be sold on a stock exchange.

Corporations raise funds to finance their operations by borrowing as well as by issuing stock. The corporation may take out a loan in order to meet short term financial needs or it may seek a mortgage to pay for a major investment project, just as a home buyer will seek a mortgage to finance the purchase of that dream house. Corporations also borrow funds by issuing bonds. A *bond* is a legally enforceable obligation to pay back the loan on maturity and to make interest payments periodically in the interim. For example, a $10,000 bond maturing in 2015 paying 6% is a promise to pay the owner of the bond $10,000 when the bond matures in 2015 plus $600 each year until then. Bonds of major corporations are actively traded in the marketplace and their current prices, determined by supply and demand, are quoted daily in the *Wall Street Journal* and other financial publications. This means that if you inherit a $10,000 bond from your late uncle you will not have to wait until it matures before you can get your money; you can sell it to someone else who is willing to hold the bond until maturity. A broker will be happy to handle the transaction for you for a fee. You will find that the going price of the bond today may be substantially more or substantially less than the $10,000 face value that it will pay on maturity, depending on market conditions.

Owners versus creditors

There is a fundamental distinction between the firm's owners and its creditors. *Debtors* are private individuals or organizations that have borrowed money. A student using a credit card and a homeowner paying off a mortgage on their home are debtors. Corporations are debtors because of the funds they owe to bankers and bondholders. The *creditors* are those who have loaned the money. The banker, the insurance company or a private individual buying the corporation's bonds or lending it funds receives a commitment from the borrower to pay off the loan at a specific date. The borrower also has a contractual obligation to make regular interest payments on the loan. The firm's owners — its stockholders — have no such guarantee even though they also have advanced funds to help finance the firm's investments and operations. If the firm prospers, the owners will be rewarded with a share of the firm's profits, but if the firm loses money the owners are the first to shoulder the loss. The lenders must be paid on time before the owners get anything. Creditors lose only if business is so bad that there is not enough to fully pay off all the outstanding loans.

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When a firm cannot meet its commitments to its creditors, *bankruptcy* may follow. Bankruptcy may be declared by the firm itself or initiated by disgruntled creditors. Because the United States Constitution grants Congress the authority to "establish ... uniform laws on the subject of Bankruptcy throughout the United States," bankruptcy proceedings are supervised by and litigated in the United States Bankruptcy Courts rather than in state courts. There are two types of bankruptcy: If the business enterprise is liquidated, a court appointed trustee will sell off the assets of the firm and distribute the proceeds to the creditors in proportion to the amount they are owned — this might be 90¢ or 10¢ for each dollar of debt, depending on how much is realized from the sale. Alternatively, the courts may allow the debtor firm to continue in business in an effort to rehabilitate itself and, it is hoped, eventually be able to pay off its creditors.

Here are two bankruptcies that undermined investor confidence and traumatized the stock market in year 2002:

1. Enron Corporation: Late in 2001, shortly after admitting major accounting errors that inflated earnings by almost $600 million since 1994, this giant Texas-based energy trading company filed for bankruptcy. With $62.8 billion in assets, it became the largest bankruptcy case in U.S. history, up to that time. The price of Enron stock tanked, many employees lost their pensions and life savings. Arthur Andersen, one of the very largest accounting firms in the United States, was convicted of obstructing justice by shredding Enron documents and forced out of business.

2. WorldCom, the telecom giant, filed for bankruptcy in July of 2002 after reporting that its profits had been overstated by $3.8 billion in bogus accounting, and assumed from Enron the dubious distinction of suffering the largest bankruptcy in the nation's history. Subsequent investigation revealed that there had been more than $7 billion in bogus accounting. The company's former CEO and Chairman Bernard Ebbers received a low interest rate loan of $408 million dollars and $1.5 million-a-year for life in severance pay. The bankrupt firm took legal proceedings to rescind Ebbers' severance package, arguing that the severance agreement was negotiated under fraudulent terms. The company had laid off 17,000 workers earlier in the year.

Corporate Governance

The collapse of stock markets in 2001–2002 plus revelations about gross accounting improprieties at a sizable number of major corporations raised serious concern about the structure by which corporations run their affairs and the proper role for government in their regulation. Do the shareholders who own the firm have effective control? Does the company's Chief Executive Officer (CEO) run the company in the interest of the owners or to maximize personal gain? These were not new questions. In 1932, in the aftermath of the 1929 stock market crash, Adolf Berle and Gardiner Means focused in *The Modern Corporation and Private Property* on the "separation of ownership from control." Seventy years later, Alan Greenspan, the powerful Chairman of the Federal Reserve Board (the central bank of the United States), who had himself served on the boards of 15 major corporations, commented as follows on the problem:[5]

> ... as our economy has grown, and our business units have become ever larger, de facto shareholder control has diminished: Ownership has become more dispersed and few shareholders have sufficient stakes to individually influence the choice of boards of directors or chief executive officers. The vast majority of corporate share ownership is for investment, not to achieve operating control of a company. Thus, it has increasingly fallen to corporate officers, especially the chief executive officer, to guide the business, hopefully in what he or she perceives to be in the best interests of shareholders. Indeed, the boards of directors appointed by shareholders are in the overwhelming majority of cases chosen from the slate proposed by the CEO. The CEO sets the business strategy of the organization and strongly influences the choice of the accounting practices that measure the ongoing degree of success or failure of that strategy. Outside auditors are generally chosen by the CEO or by an audit committee of CEO-chosen directors. Shareholders usually perfunctorily affirm such choices.
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> To be sure, a CEO can maintain control over corporate governance only so long as companies are not demonstrably in difficulty. When companies do run into trouble, the carte blanche granted CEOs by shareholders is withdrawn. Existing shareholders, or successful hostile

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[5]Remarks by Chairman Alan Greenspan delivered at the Stern School of Business, New York University, New York, New York, March 26, 2002: http://www.stern.nyu.edu/greenspan/alan_greenspan_remarks_0326.pdf.

bidders for the corporation, usually then displace the board of directors and the CEO. Such changes in corporate leadership have been relatively rare but, more often than not, have contributed to a more-effective allocation of corporate capital.

Reform

Public outrage at the growing list of corporate scandals coupled with the plunge of the stock market led to the passage of the Sarbanes-Oxley Act in July of 2002. The reform legislation provides for the establishment of the Public Company Accounting Oversight Board to watch over the auditing of corporations. It requires that each corporation's Chief Executive Officer (CEO) and Chief Financial Officer (CFO) certify to the validity of their firm's financial statements. Each corporation must have an audit committee composed entirely of directors of the firm who are independent in the sense that they do not accept consulting advisory or other compensatory fees from the company apart from their capacity as members of the firm's board of directors. The act also increased the penalties for violating the security laws and prohibited the destruction or falsifying of records. It banned corporations from making personal loans to their executive officers and directors.

While many critics objected that the law did not go far enough, some complained that it might inhibit entrepreneurial innovation and place American enterprises at a disadvantage in competing with foreign firms. The continuing decline in the stock market in the months immediately following the passage of the act implied that the reform legislation failed to restore investor confidence in the integrity of the system.

5.3   Profits and accounting

Accounting records report the financial position of the business enterprise, providing management with information about the business that is essential for making wise decisions. Accountants also generate reports for use by those outside of the firm, such as the financial reports for shareholders and potential investors, including the tables that appear in the corporation's annual report. Before banks will make a loan, they appraise the creditworthiness of the borrower and carefully scrutinize the accounting records of the business in order to learn how much profit the firm has been making and how much the business is worth today. And of course, the Internal

Revenue Service requires corporations to file tax returns reporting their profits and related information.

The Securities and Exchange Commission was created in 1934 to establish financial accounting and reporting standards for publicly held corporations. Since 1973 the responsibility for establishing financial accounting standards has been delegated to the Financial Accounting Standards Board, a private organization. This organization promulgates the *Generally Accepted Accounting Principles* (GAAP),which are officially recognized as authoritative by both the Securities and Exchange Commission and the American Institute of Certified Public Accountants. Nonetheless, GAAP leave considerable room for the firm's accountants to exercise discretion in preparing the books. In deciding such arcane issues as how to measure depreciation and what inventory accounting procedures to employ, the accountant has a certain amount of leeway to manage the profit picture that will be reported on the firm's books. Accountants may be encouraged by management to adopt strategies that puff reported profits in order that the firm's stock will catch the eyes of investors, push up the value of the stock, and possibly lead to a bonus for the corporation's president. For tax returns the objective is just the opposite. The accountant may adopt tax-reduction strategies that will minimize profits that have to be reported for tax purposes.

5.3.1 *The balance sheet*

A firm's *balance sheet* provides a picture of the financial posture of the enterprise at a particular date. There are three major items on the balance sheet:

- *Assets*: An asset is something of value owned by the firm, such as its holdings of land, buildings and machinery. Inventories of raw material waiting to be processed and finished goods that are unsold or awaiting shipment to customers are also counted among the assets of the firm. Cash in the safe and funds on deposit in the bank are also assets. So are patents and copyrights. If the firm has accounts receivable from its customers, they are listed among the firm's assets.
- *Liabilities*: In accounting, liabilities are what the enterprise owes to its creditors. Included are funds owed to the banks, the value of bonds outstanding, accounts payable to suppliers, and taxes owed to the Internal Revenue Service.

- *New Worth*: The net worth of an enterprise is the accountant's estimate of the value of the firm to its owners. It is the excess of the assets of the firm over its liabilities.

The fundamental equation of accounting states the relationship between these three concepts:

$$\text{Net Worth} = \text{Assets} - \text{Liabilities} \tag{1}$$

As this equation reveals, Net Worth is a residual. It is what is left over for the owners of the firm after the claims of all the creditors are met. Note that the firm's creditors are those who have lent funds (i.e., granted credit) to the firm. Stockholders are not creditors; they are the owners of the corporation, sharing in the profits or losses and, at least in principle, exercising control over the enterprise.

Now let us consider the simplified balance sheet for a hypothetical firm displayed on Table 5.2. First of all, note that the Assets of the firm are listed on the left side of the balance sheet and its Liabilities and Net Worth are on the right. This is in conformity with the following modification of equation (1):

$$\text{Assets} = \text{Liabilities} + \text{Net Worth} \tag{2}$$

Thus the sums of the two sides of the table must add to the same number, and indeed they do, $1,550,000!

The accountant's treatment of plant and equipment requires explanation. The $800,000 listing for plant and equipment is the amount that the firm paid for the various items on the dates when it purchased them; it is their *historic cost* rather than replacement cost. Because the plant and

Table 5.2.   Balance sheet of the Fly-by-Nite Aircraft Company.

December 31, 2000 (All figures in $1,000)				
Assets			*Liabilities*	
Cash	150		Accounts payable	50
Inventory	500		Bonds	350
Accounts receivable	300		Total liabilities	400
Plant and equipment	800		Net worth	
less depreciation	(200)		Common stock	1,000
	600		Retained earnings	150
			Total net worth	1,150
Total Assets	$1,550		*Liabilities + net worth*	$1,550

equipment are far from new; they are not as valuable as they were when purchased. The *depreciation* item of $200,000 is the accountant's estimate of an appropriate allowance for the wear and tear on the plant and equipment.[6] The number is placed in parentheses, which is the accountant's way of indicating that it is to be subtracted. Subtracting the depreciation from the original purchase price yields the accountant's estimate of the current value of the plant and equipment. The final Plant and Equipment figure of $600,000 is the value of factory buildings and equipment at historic cost adjusted for wear and tear. But the figures have not been adjusted for inflation, which means that they may grossly understate the cost of replacement.

Liabilities appear as the first heading on the right hand side of the balance sheet. As of December 31, 2000, our firm owed $400,000 to its suppliers and bond holders.

The Net Worth item on the right side of the balance sheet requires further explanation. Net Worth is a *residual*. It is not apparent from the balance sheet, but as equation (1) made clear, net worth is obtained by subtracting Liabilities from Assets — this gives us the Total Net Worth entry of $1,150,000. The *retained earnings* entry is the sum of profits that, over the years, have been plowed back into the firm rather than distributed to the owners as dividends. The entry for Common Stock is the total net worth entry minus the retained earnings.

The balance sheet is a statement about the firm's financial position at a particular point of time, in this case December 31, 2000. The balance sheet does not tell us how much in the way of profit the firm has made during the year or how much it has been paying out in dividends. Profits and dividend payments are examples of flows that take place over a period of time. Just as a snapshot cannot accurately reveal the speed of a cross-country runner, so too, a balance sheet cannot reveal profits. A second type of accounting record, the income statement (aka a profit and loss statement) reports on what happened to the firm over a period of time, just as a video camera may vividly reveal the speed of our cross-country runner.

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[6]How depreciation is calculated is a complicated topic that will deserve further discussion a bit later in this chapter.

Table 5.3.   Income statement of the Fly-by-Nite Aircraft Company.

January 1, 2001 to December 31, 2001 (all figures in $1,000)				
Net sales			800	
Less Cost of manufacturing				
Materials	200			
Labor	250			
Depreciation expense	100			
		550		
Less inventory increase		(100)		
Cost of goods sold		(450)		
Gross margin			350	
Less selling cost		30		
Interest expense		20		
			(50)	
Profits				300
Less corporate profit tax				(105)
Profits after taxes				195
Less dividends				(100)
Addition to retained earnings				95

5.3.2   *The income statement*

Suppose that during year 2001 the following transactions are recorded on the books of the Fly-by-Nite Aircraft Company (in $1,000): Materials Purchased $200; Labor costs $250; Sales $800; Selling Cost $30; Dividends $100; Depreciation Expense, $100; Bond interest, $20; Corporate Profits Tax, $105. Further, as of December 31, 2001, Accounts Payable were $100, Accounts Receivable $200, Inventories $600. How can we determine the firm's profits from this information? Table 5.3, the income statement, lays out the information in a way that makes it easy to see what has been going on. Here are some key points to note about the income statement:

- The income statement summarizes flows that took place during year 2001. We shall find that these flows will explain why entries in the end of year 2001 balance sheet differs from the year 2000 balance sheet.
- Observe that inventories have increased during the year. That is to say, part of the expenditures of the firm on purchased materials and labor costs were incurred in accumulating stocks of purchased materials and/or finished product that will be available for future use. These expenses should not be counted as part of cost of goods sold during the current year.

- Interest costs (which are income to the firm's creditors) are an expense that can be subtracted before computing the firm's profits, which are subject to the corporate profit tax.
- The corporation pays dividends from after-tax profits, but only part of after-tax profits are distributed as dividends to the owners of the firm.
- The addition to retained earnings of $95 constitutes funds kept back by the firm for future expansion rather than distributed to the stockholders. Stockholders may expect eventual rewards whenever they sell their stock because it should sell for more, thanks to the investment in plant and equipment financed from retained earnings.

5.3.3   *Taxing complications*

Capital gains

Suppose you had purchased 100 shares of Fly-by-Nite Aircraft for $10 a share on March 10, 1996. By December 31, 2000, Fly-by-Nite stock was selling for $15 a share. Thus the value of your holdings had increased from $1,000 to $1,500. This increase in the value of the shares is known as a capital gain, but because you have not sold the stock it is an *unrealized capital gain.* Unrealized capital gains are not taxable. If you sold the stock on July 1 for, say $14 a share you will have *realized* a capital gain. The excess of what you sell your stock for over what you paid for it, $1,400 − $1000 = $400 is your *realized capital gain*, which is taxable. You will be pleased to find when you complete your income tax return the following April that yours is a long-term capital gain because you had owned the asset for more than a year. This means that your capital gain will be taxed at a maximum rate of 15% (or 5% if your income is so low that you are in the 15% income tax bracket). If you had sold the asset for less than you paid for it, you would have realized a capital loss, which would lead to a reduction in your taxes for that year.

Why should investors pay a lower tax rate on realized capital gains than a worker pays on earned income? This hardly seems fair, but there are two arguments for the favorable treatment of capital gains. First of all, the realized capital gain may be partially offset by inflation. Suppose that during the period you owned the stock the consumer price index had increased by 10%. Then the $1,400 you receive when you sell your asset is worth only as much purchasing power as $1,400/1.1 = $1,272.72 when you purchased the asset, which means that your capital gain net of inflation,

i.e., your *real capital gain*, is only $272, but you will be taxed on the $400 nominal capital gain. The favorable treatment of capital gains provides an imprecise adjustment for inflation. The second argument for favorable taxation of capital gains is that the taxation of realized capital gains has a *lock-in effect*. Faced with the prospect of a substantial capital gains tax if they sell their assets, investors may decide to hang on to stocks that they would otherwise sell. Taxing capital gains at a reduced rate is one way of limiting the strength of the lock-in effect.

Step-up-of-basis-at-death

Those who invoke the "lock in effect" to justify the favorable tax treatment of capital gains do not always champion the elimination of another type of "lock in effect." If you hold your stock until you die, both you and your heirs will avoid the capital gains tax! The "step-up-of-basis on death" provision of the internal revenue code allows an heir selling an inherited asset to compute the realized capital gain on the basis of the increase in the value of the asset from when it was inherited rather than its price when initially purchased by the deceased. This means that no tax is ever paid on the gain in value that took place during the period in which the asset was owned by the deceased.

Double taxation

Stock market investors often complain that they are unfairly subjected to double taxation. First, the corporation in which they have purchased stock has to pay the corporate profits tax. Second, they have to pay income tax on the dividends they receive from the corporation. Thus, their investment earnings are taxed twice. Workers may sympathize with the stockholders' lament because they have to pay both the income tax and the Social Security tax on their wage income.

The actual burden of *double taxation* of corporate profits is not quite as severe as it appears for two reasons. First of all, only that part of corporate profits that is paid out as dividends is taxable income for the investor. Corporate retained earnings are not immediately taxable. The hope is that the shares will increase in value because the funds are being put to good use by the corporation. Thus, the personal income tax on the retained portion of corporate profits is postponed until the capital gains are realized when the stock is sold. And when the return is finally taxed it is at a much lower rate than wage income. Second, endowment investments

5.4.2   *The law of diminishing returns*

The Law of Diminishing Returns is the proposition that the additional output from successive increases of one input will eventually diminish, provided all other inputs are held constant. This "law," stressed as early as 1817 by David Ricardo, has been regarded as of fundamental importance by generations of economists. Point a on Figure 5.4 and Figure 5.5 marks the point where additional labor starts to reduce the average product of labor. Point *m* marks the *point of diminishing marginal productivity.*

Two exceptions to the law of diminishing returns

While the proposition may initially appear obvious, here are two exceptions that deserve consideration.

1. *Pipeline*: When building an irrigation pipeline, a major cost factor is the steel for the pipe. The number of square feet of steel required per running foot of pipeline obviously depends on the circumference of the pipe: $C = 2\pi r$, where r is the radius of the pipe. The capacity of the pipe in gallons per hour, given the pumping pressure, will be proportional to the cross-section area of the pipe, which is a circle with area $A = \pi r^2$. Thus the cost/capacity ratio will be proportional to $A/C = 2/r$.[8] This ratio, and hence the steel required per unit of output declines with output, violating the Law of Diminishing Returns.

2. *Networks*: Telephone, faxes and the Internet are all examples of *networks*, and they all violate the law of diminishing returns. Their common feature is that the value of the network to any user depends positively on the number of users. If you were the only subscriber to the local telephone company, the service would be of no value because there would be no one to talk to. With two telephone subscribers you can call one person, with three two, and so forth. If x is the average value to you of being able to contact someone, then the value to you of the telephone is $(n-1)x$, where n is the number of subscribers (including you). The value of a network to the n users is $v(n) = n(n-1)x = (n^2 - n)x$. This demonstrates "Metcalfe's law," the proposition that the value of a network is proportional to the square of the number of connected clients.[9]

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[8]For a more elaborate discussion, see Hollis B. Chenery, "Engineering Production Functions," *Quarterly Journal of Economics*, 1949.
[9]Robert Metcalfe founded 3Com Corporation and designed the Ethernet protocol for computer networks.

> So too with Email, the fax machine and any other innovation where the value to each user of the device is proportional to the number of other users one can contact.

The concept of networking explains why many innovations are slow to catch on. It does not pay to join the network when there are few users. But once the number of users reaches a critical mass, the rewards clearly outweigh the cost, and more and more users rush to take advantage of the innovation.

5.4.3   *Multiple inputs and the production function*

A natural extension of the total product curve is to include more than one input, which brings us to the concept of the production function. The *production function* is like a recipe book in that it shows the level of output that can be obtained by applying various combinations of labor, machinery and equipment, raw materials and other inputs.

While in reality the typical firm uses a wide range of resources in producing a number of different products, it is appropriate to narrow our attention to the essential features of the problem by supposing that our hypothetical firm produces one type of product utilizing only two inputs, labor and capital.[10] Here we mean by *capital* the machinery, buildings, equipment and other durable items that are available for use in the production process.[11] In reality there are many kinds of labor and types of capital employed by a business enterprise, but we shall neglect these complications and assume that we can talk meaningfully about the quantity of labor and capital services that are employed by the firm. Thus we shall write our production function as

$$q = q(L, K). \tag{5}$$

The production function is analogous to the utility function we used in building the theory of the consumer in Chapter 4. But a major difference is that we can measure the output the firm produces with a given set of resources while in Chapter 4 it was necessary to concede that we cannot directly measure the utility realized by the consumer.

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[10]Students who have studied multivariate calculus will find that the analysis that follows may be easily extended from 2 to any finite number of inputs.

[11]The term "capital" is used in two senses. Here we are measuring the stock of buildings and durable physical equipment. But the term capital is used in both accounting and finance to refer to the total funds that have been contributed by the owners of the firm to finance their business enterprise.

A simple example is provided by the Cobb-Douglas production function:

$$q = q(L, K) = \alpha L^{\lambda} K^{\lambda'}, \quad L \geq 0, \; k \geq 0, \; \lambda \geq 0, \lambda' \geq 0. \qquad (6)$$

Note that in the Cobb-Douglas case $q(0, K) = q(L, 0) = 0$; i.e., both labor and capital are required if anything is to be produced.

This production function is named after economist Paul H. Douglas [1892–1976] and C. W. Cobb, his math department colleague at Amherst College.[12] Utilizing annual data on industrial production in the United States covering the years 1899–1922, they estimated:[13]

$$q(L, K) = 0.0156 L^{0.807} K^{0.232} ; \qquad (7)$$

i.e., $\alpha = 0.0156$, $\lambda = 0.807$ and $\lambda' = 0.232$. This empirical investigation established Douglas's reputation as one of the leading economists of the day.[14]

The iso-product curves on Figure 5.6 provide an informative contour plot of the production function, which is analogous to the indifference maps of Chapter 4. The quantity of labor is plotted on the abscissa and the quantity of capital on the ordinate. The parameter values are $\alpha = 2$, $\lambda = 2/3$ and $\lambda' = 1/3$. Each *iso-product curve* (aka "isoquant") reveals the various combinations of the two inputs that would just suffice to produce the specified level of output. Points a, b, c and d on the light horizontal line at $K = 10$ indicate how the level of output would respond to increasing quantities of labor, keeping the stock of capital fixed at $K = 10$. These same three points are also plotted on the total product curve for $K = 10$ on Figure 5.7, where the equation for the total product curve is $q(L, 10) = 2L^{2/3} 10^{1/3}$.

For the Cobb-Douglas production function we have as the average product of labor

$$\frac{q}{L} = \alpha L^{\lambda - 1} K^{\lambda'} . \qquad (8)$$

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[12] As often happens in the history of science, the Cobb-Douglas function is misnamed. Although the theoretical properties of the production function they used had already been studied by Philip Wicksteed [1844–1927], this function is universally known as the Cobb-Douglas.

[13] Taking logs of both sides of equation (6), they calculated without the aid of computer the values of the unknown parameters λ and λ' and α that would minimize $\sum e_t^2$ in the equation $\log q_t = \alpha + \lambda \log L_t + \lambda' \log K_t + e_t$. This simple estimation procedure is known as the "method of least squares."

[14] Douglas taught at the University of Chicago as well as Amherst. After service in the Marines during World War II, he was elected to three terms in the United States Senate, where he chaired the Joint Economic Committee.
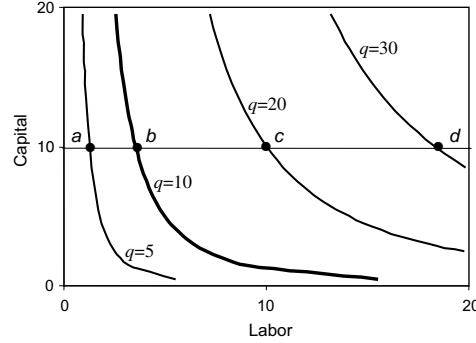
Fig. 5.6.   Iso product curves, Cobb-Douglas production function
The iso-product curves show the combinations of capital and labor that will yield the specified level of output. Points a, b, c and d show the quantity of labor required (read off the abscissa) to obtain the specified level of output, given that $K = 10$. The production function is Cobb-Douglas: $q = 2L^{2/3}K^{1/3}$.
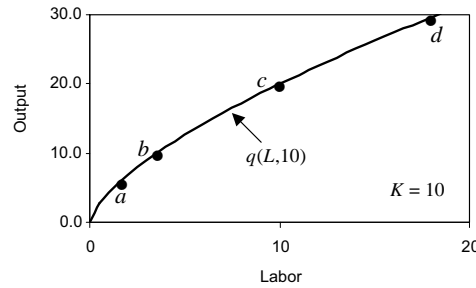

Fig. 5.7.   Total product curve ($K = 10$)
The total product curve, derived from the iso-product graph, plots output as a function of labor, given the specified quantity of capital. Points a, b, c and d are carried over from Figure 5.6.

Unlike Figure 5.3, this total product curve does not display an area of increasing returns because the Cobb-Douglas functional form is so simple.

The marginal product of labor is

$$\frac{\partial q}{\partial L} = \lambda \alpha L^{\lambda-1}K^{\lambda'} \,, \tag{9}$$

which simplifies to

$$\frac{\partial q}{\partial L} = \frac{\lambda \alpha L^{\lambda}K^{\lambda'}}{L} = \frac{\lambda[\alpha L^{\lambda-1}K^{\lambda}]}{L} = \frac{\lambda q}{L} \,. \tag{10}$$
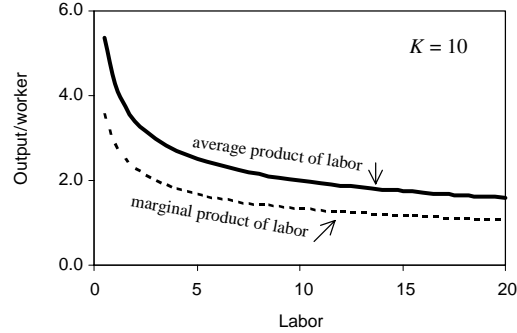
Fig. 5.8.   Average and marginal product of labor (Cobb–Douglas case)
For the special case of the Cobb–Douglas production function, marginal product $(\partial q/\partial L)$ is proportional to the average product of labor (Q/L); specifically, $\partial q/\partial L = \lambda Q/L$.

The average and marginal product of labor are plotted on Figure 5.8 for the case of 10 machines ($K = 10$), $\lambda = 2/3$ and $\lambda' = 1/3$. These are simpler than those on Figure 5.4 because the Cobb–Douglas production function is subject everywhere to diminishing returns.

The production function provides a simple but useful summary of the available technology. Our next task will be to put it to work in generating cost functions.

5.5   Maximizing profit

The profit maximizing level of output for the representative firm is determined by the appropriate balancing of revenue and costs. We will show how the firm's cost function may be derived from its production function. Then we will seek the profit maximizing level of output.

5.5.1   *The cost function*

The firm's *cost function* explains how its cost of production depends on the level of output. The *short-run cost function* shows how production costs depend on output, given a fixed number of machines, $\overline{K}$, where the bar over the K indicates that it is to be regarded as fixed. Figure 5.8 showed how output depended on the number of workers, given $\overline{K} = 10$. Suppose we are told that it costs $r = \$5$ per hour to use a machine and the workers

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get a wage $w = \$10$ per hour. Then total costs will be

$$C = r\overline{K} + wL = 5\overline{K} + 10L\,. \tag{11}$$

But this accounting relationship is not the total cost function because it does not relate costs to output. To find the short-run total cost function $C(q,\overline{K})$ showing the minimum cost of producing the specified output, given $\overline{K}$, we need to harness the technological information provided by the production function. We must obtain from the production function the inverse function showing the minimum quantity of labor required to produce q units of output, given $\overline{K}$. This required quantity of labor function, call it $L^*(q,\overline{K})$, is in the case of the Cobb-Douglas production function (6)

$$L^*(q,\overline{K}) = \alpha^{-1/\lambda}q^{1/\lambda}\overline{K}^{-\lambda'/\lambda}\,. \tag{12}$$

This is not as complicated as it seems, for $L^*(q,\overline{K})$ is just the inverse of the total product of labor curve plotted on Figure 5.7 for $\overline{K} = 10$. Substitution of $L^*(q,\overline{K})$ into (11) yields the cost of producing q units of output, given $\overline{K}$:

$$\begin{aligned} C(q,\overline{K}) &= wL^*(q,\overline{K}) + r\overline{K} \\[2mm] &= w\alpha^{-1/\lambda}q^{1/\lambda}\overline{K}^{-\lambda'/\lambda} + r\overline{K} \end{aligned} \tag{13}$$

From this short-run total cost function we find that in the short run (given $\overline{K}$) the average total cost is

$$\frac{C}{q} = w\alpha^{-1/\lambda}q^{(1-\lambda)/\lambda}\overline{K}^{-\lambda'/\lambda} + \frac{r\overline{K}}{q}\,. \tag{14}$$

Differentiating (13) yields marginal cost:

$$\frac{dC(q,\overline{K})}{dq} = \frac{w\alpha^{-1/\lambda}q^{(1-\lambda)/\lambda}\overline{K}^{-\lambda/\lambda}}{\lambda} = \frac{[C(q,\overline{K}) - rK]}{\lambda q}\,. \tag{15}$$

Average and marginal costs are plotted on Figure 5.9. The total costs of producing various levels of output per hour are summarized on Table 5.4 for parameter values $\lambda = 2/3$ and $\lambda' = 1/3$; the wage is $w = 20$ and the cost of using a machine is $= \$5$.

Because the Cobb Douglas is a particularly simple form of the production function it generates quite simple cost functions. While the Cobb

Douglas case will suffice for many purposes, in Chapter 6 we will find it necessary to consider a somewhat more complicated shape for the cost function. In general, given any production function $q(L, K)$, the short-run total cost function is

$$C(q, \overline{K}) = r\overline{K} + wL^*(q, \overline{K}), \tag{16}$$

where $L^*(q, \overline{K})$ is the required quantity of labor function derived from $q(L, K)$.
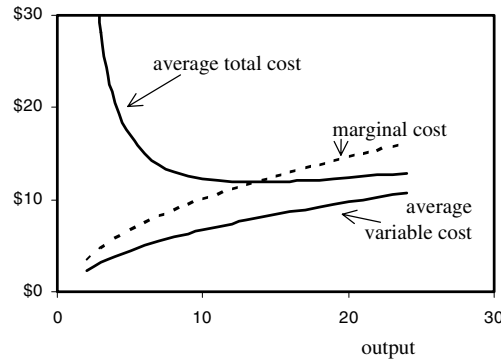

Fig. 5.9.   Short-run cost functions
These costs functions were derived from our Cobb-Douglas production function under the assumption that $r = 5$, $w = 20$ and $\overline{K} = 8$.

Note that at the level of output where average total cost is at its minimum, marginal cost equals average total cost.

Table 5.4.   Shortrun total cost function.

Output	Labor required	Variable cost	Total cost	Average cost	Average variable cost	Marginal cost
q	$L^*(q)$	$C(q) - r\overline{K}$	$C(q)$	$C(q)/q$	$(C(q) - rK)/q$	$dC(q)/dq$
0	0.0	0.0	40.0			
5	1.4	28.0	68.0	13.6	5.6	8.4
10	3.9	79.1	119.1	11.9	7.9	11.9
15	7.3	145.2	185.2	12.3	9.7	14.5
20	11.2	223.6	263.6	13.2	11.2	16.8
25	15.6	312.5	352.5	14.1	12.5	18.8

5.5.2 *Short-run profit maximization*

The total profit of our firm equals total revenue minus total costs.

$$\boldsymbol{\pi}(q) = R(q) - C(q)\,, \tag{17}$$

where $R(q)$ is total revenue, price times quantity. (We may simplify notation by omitting the fixed capital stock $\overline{K}$ from the total cost function, writing $C(q)$ for total costs.) If our firm sells its product on a competitive market, it will be a price taker, the price it receives for its product being independent of how much it desires to sell. If our firm is a monopoly, however, the price it will receive for its product will depend on the quantity it decides to sell. Quite generally, we may specify total revenue to be

$$R(q) = p(q)q\,, \tag{18}$$

where $p(q) = q^{-1}(p)$ is the inverse demand function discussed in Chapter 3. In the competitive case, $dp/dq = 0$ and so the demand curve is horizontal. Under monopoly, $dp/dq < 0$, the demand curve having a negative slope. To find the q that maximizes profits we set the first derivative of profit function (17) equal to zero:

$$\frac{d\boldsymbol{\pi}(q)}{dq} = \frac{dR(q)}{dq} - \frac{dC(q)}{dq} = 0\,. \tag{19}$$

Therefore, profit maximizing requires

$$\frac{dR(q)}{dq} = \frac{dC(q)}{dq}\,. \tag{20}$$

This is the fundamental proposition that in order to maximize profits it is necessary to adjust output so that *marginal revenue*, (dR/dq), equals *marginal cost*, (dC/dq).

Since marginal revenue is a concept of fundamental importance, it deserves closer inspection. Differentiation of (18) reveals that marginal revenue is

$$\frac{dR}{dq} = p + \frac{dp}{dq}q\,. \tag{21}$$

In the case of competition, $dp/dq = 0$ and marginal revenue equals price. The monopoly case is clarified on Figure 5.10. From equation (21) it is clear that at $q = 0$, $dR/dq = p$, as indicated on the graph. Also, if as on the graph the demand curve is linear, the slope of the marginal revenue curve will be $d^2R/dq^2 - 2\,dp/dq$; i.e., the marginal revenue curve is twice
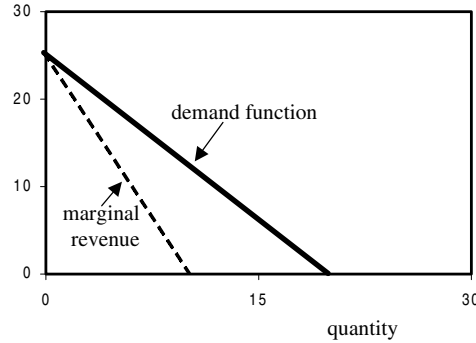
Fig. 5.10.   Demand function $q = 20 - (4/5)p$
Since the inverse demand function is $p(q) = 25 - (5/4)q$, revenue is $R(q) = qp(q) = 25q - (5/4)q^2$ and marginal revenue is $dR(q)/dq = 25 - (5/2)q$.

as steep as the demand curve. All this means that whendemand is linear the marginal revenue line will cross the abscissa at $q(0)/2$.

   There is an interesting relationship between marginal revenue and elasticity:

$$\frac{dR}{dq} = p + \frac{dp}{dq}q = p\left(1 + \frac{dp}{dq}\frac{q}{p}\right) = p\left(1 - \frac{1}{\eta}\right), \tag{22}$$

where $\eta = -(dq/dp)(p/q)$ is the price elasticity of demand discussed in Chapter 3.

   Note the following two fundamental points:

#1. Because a firm in a competitive industry is a price taker facing a horizontal demand curve, $dp/dq = 0$ and so marginal revenue equals price.

#2. If demand is inelastic, $(\eta < 1)$, then $dR/dq < 0$.

The second proposition implies that no profit maximizing firm would sell at a point where its demand elasticity is less than one. Why? Because by selling less it could increase revenue without increasing total cost.

   The profit maximization story for our firm is summarized graphically on Figure 5.11. The average and marginal revenue curves have been reproduced from the preceding graph and the average and marginal cost curves from Figure 5.8. Profits are maximized at the output level q for this yields marginal revenue equal to marginal cost at point e. The highest price that can be obtained at this level of output is p. Average cost at output q, call it c_a, may be read off the average total cost curve. Profit per unit of
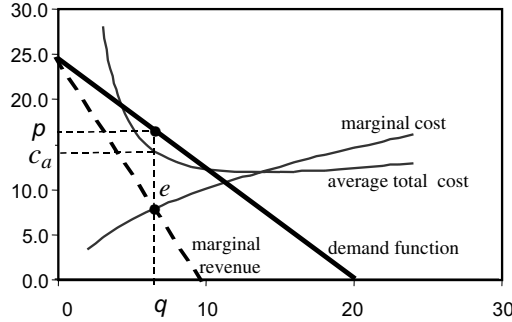
Fig. 5.11.   Profit maximization ($\overline{K} = 10$)
The profit maximizing level of output can be determined once the demand and marginal
revenue curves of Figure 5.10 are superimposed on top of the marginal and average total
cost functions of Figure 5.9.

Profits are maximized by setting output at q because at point e marginal revenue
equals marginal cost. Price p is the maximum price at which q can be sold.

output is the excess of price over average cost, $p - C_a$, and total profits of
$\pi = (p - C_a)q$ are indicted by the upper rectangle on the graph.

5.5.3   *Least-cost input mix — Costs in the long run*

The preceding analysis relied on the assumption that the timeframe for
the analysis was so short that our firm could only adjust the labor input;
there was insufficient time to adjust the quantity of physical capital (e.g.,
machinery and factory space) — that is what is meant by the *short run*. In
the *long run*, the firm has enough time to adjust the number of machines
as well as the number of workers. Our firm must adjust its capacity on the
basis of its projections of future demand for its product.

As a first step toward deriving the firm's long-run cost function, suppose
our firm expects to sell 10 units of output. What would be the best input
mix? That is to say, what combination of capital and labor will minimize
the total cost of producing this level of output? Our firm's task is to
minimize

$$C(L, K) = wL + rK \qquad (23)$$

subject to the constraint

$$q(L, K) = 10 \,. \qquad (24)$$