Optimal Lot Size, Inventories, Prices and JIT Under Monopolistic Competition*

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Abstract

This paper attempts to demonstrate that the disciplines of Operations Research/Management Science and economics are complementary by drawing on both in analyzing the effects of the introduction of Just in Time on prices and product variety.

From the discipline of OR/MS I draw on the Optimal Lot Size Model. From the discipline of economics I borrow the theory of monopolistic competition. The model of monopolistic competition recognizes the significance of product differentiation: Because no two firms in a monopolistic competitive industry produce exactly the same product, the demand curve confronting each producer is downward sloping, like monopoly. Because of free entry and exit, economic profits are driven to zero in the long run, like perfect competition. This paper introduces inventories into the model of monopolistic competition by incorporating optimized setup and inventory carrying costs into the representative firm's total cost function.

I show that the adoption of Just in Time (JIT), implemented by reengineering the product so as to drastically reduce or eliminate setup costs, benefits consumers, at least in the long run: competitive forces mean that the adoption of JIT leads to both a reduction in price and an increase in the variety of products offered in the marketplace. But alas, the rewards to the innovating business firm are only transitory. Free entry drives economic profits back to zero for both the innovating and the imitating firms.

MS/OR + Economics

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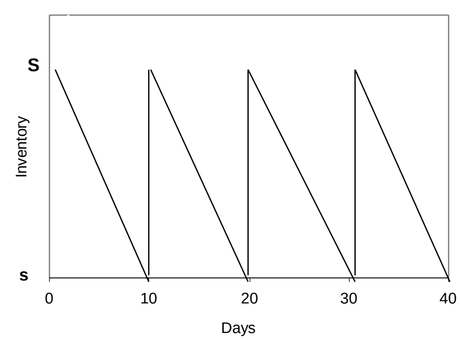
1. Introduction

Figure 1: S-s Inventories



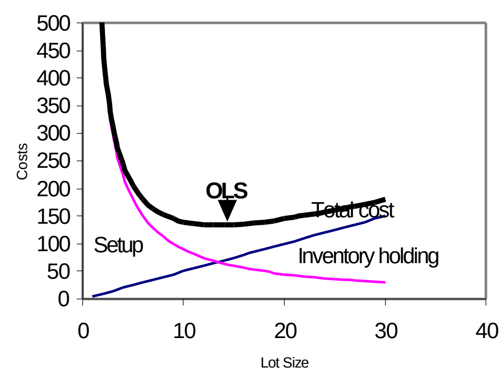
Practitioners of Operations Research and Management Science make their living by telling business firms how they can operate more profitably, and they pass the market test. Economists customarily assume that business enterprises succeed in maximizing profit. While economists thus assume that the Operations Researchers have completed their task, I think that much may be gained by integrating these two disciplines. I think it unfortunate that economists don't try to learn more from our sister disciplines.[1]

In this paper, I will try to demonstrate that these are indeed complementary disciplines by merging two distinct strands of thought: From the discipline of Operations Research I draw on the concept of OLS (Optimal Lot Size not ordinary least squares). From micro economic theory, I draw on the theory of Monopolistic Competition. After a brief review of the relevant features of these two theories, I will combine them into a model that can be used to analyze certain economic consequences of adopting *Just-in-Time*. I investigate how prices and the number of firms in the industry will adjust as a result of adopting this new management style.

2. Two Models

2.1. Classic Optimal Lot Size (OLS) Model

Figure 2: Optimal Lot Size



Our firm produces its product in lots (or batches) of size D in meeting annual sales q. The goods produced in each batch are placed in inventory and gradually sold off. After a batch is sold off and inventory reduced to zero, another lot is produced and the cycle repeats. The resulting saw-tooth movement of the inventory stock is displayed on Figure 1 Two types of cost must be balanced in deciding how big a batch D to produce:

1. It costs money to set up the machinery to produce a batch and there may be cleanup or shutdown costs at the end of the run.
2. If we produce larger batches, we will have fewer setup costs each year and our total annual setup cost will be less. But the bigger the batches the larger our inventory, and holding inventory costs us money.

[1] A major contribution of ISIR is to provide a venue for such interaction, as at the 1994 Lake Balaton Workshop on Micro Foundations of Macroeconomic Analysis of Inventories.

The *optimal lot size* minimizes the sum of these two types of cost. Let q equal annual sales and D the size of a batch. Then there will be q/D setups a year. If c_s is the cost of a setup, then total setup costs for the year will be $c_s q/D$. Assuming sales take place at a roughly uniform rate throughout the year, inventory will range from 0 to D with an average level of D/2. If the cost of carrying a unit of output in inventory for a year is c_i, then yearly inventory cost will be $c_i D/2$. If we let k_0 denote fixed costs and k_1 the cost of the labor and materials and so forth that are required to produce a unit of output, then our total costs will be

$$C = k_0 + k_1 q + c_i D/2 + c_s q/D. \qquad (1)$$

The optimal lot size is that D which minimizes the annual costs of meeting sales demand q. The standard textbook graph is reproduced as Figure 2. Differentiating (1), we have as a necessary condition for a maximum

$$\partial C/\partial D = c_i/2 - c_s q/D^2 = 0, \text{ or}$$

$$D = \sqrt{2c_s q / c_i}\ . \qquad (2)$$

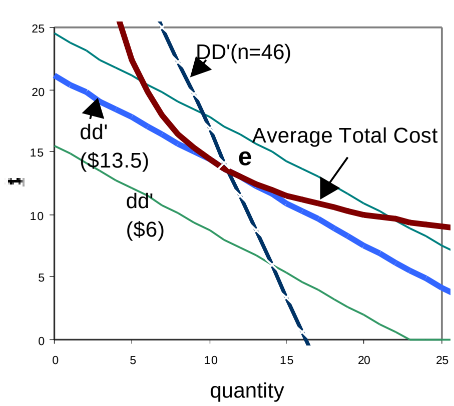
This is the famous square-root rule for the optimal lot size.[2] While the rule is famous, which is why I use it in this paper, it may not be the best choice; Linda G. Sprague and John G. Wacker[1996] explain why it is not used that much in practice.

2.2. Monopolistic Competition

The theory of monopolistic competition was advanced independently seven decades ago on different sides of the Atlantic by Edward Chamberlin [1933] and Joan Robinson [1933]. Their key contribution was to recognize that even when there are a large number of firms in an industry, the demand curve facing each firm will be downward sloping because of product differentiation, just like monopoly. But at the same time, the theory encompasses the case in which there is free entry and exit into the industry, which means that economic profit (net of the costs of owner supplied capital and labor) will be driven in the limit to zero, just like pure competition.

The standard textbook graph of a representative firm in monopolistic competitive equilibrium is presented as Figure 3. The demand curve facing the individual firm, labeled dd', is downward sloping because of product differentiation. If our firm raises its price, while other firms keep theirs fixed, our firm will loose some sales; but unlike perfect competition, it will retain some loyal customers with strong preferences for their product. Perfect competition's *Law of One Price* does not hold true under monopolistic competition, at least in the short run. The steeper DD' curve shows how sales would adjust if *all* firms in the industry were to charge the

Figure 3: Monopolistic Competition



[2] The equivalent concept, "efficient order quantity," arises in procurement. If a retailer orders items in larger quantities it will incur lower ordering costs each year, but its inventories will on average be larger. Precisely the same square root relationship arises from this argument; equation (2) holds with c_s now representing ordering costs.

identical price. But it is assumed that there are enough other firms in the industry to make it reasonable for each firm to assume that the others will not respond when it changes its own price.[3] The graph is drawn for the case of long run equilibrium – the number of firms has adjusted through free entry and exit until economic profit is zero at point e. This is the best the firm can do, but it just breaks even.

3. Synthesis

As a first step toward a synthesis we must determine the firm's total cost function, given that whatever q turns out to be our firm will use the optimal lot size specified by (2). It is easily shown that annual inventory carrying costs will be $c_iD/2 = [c_ic_sq/2]^{1/2}$, and that this also equals the costs of q/D^0 setups during the year. Substituting back into (1) yields the total cost function, given that the optimal order quantity specified by (2) is used:[4]

$$C = k_0 + k_1q + (2c_ic_sq)^{1/2}. \qquad (3)$$

Letting $\alpha = (2c_ic_s)^{1/2}$ we may write our total cost function as

$$C = k_0 + k_1q + \alpha q^{1/2}. \qquad (4)$$

We have average costs

$$C/q = k_o/q + k_1 + \alpha q^{-1/2}. \qquad (5)$$

The average cost function on Figure 2 was drawn for $k_0 = 64$, $k_1 = 4$, $c_i = 10$ and $c_s = 75$; marginal cost is

$$dC/dq = k_1 + \alpha q^{-1/2}/2. \qquad (6)$$

Our task is to work out the market implications of this type of cost function. We shall suppose that the product is sold in a monopolistically competitive market with inverse demand function

$$p_i(q_i, \overline{p}_i, n) = d_1 + d_2\,\overline{p}_i - q_in^{1/2}/d_0, \qquad (7)$$

where n is the number of firms in the industry.[5] This function implies that an increase in the number of competitive firms (i.e., a larger n) will reduce the quantity that our firm can sell at any given price. Also, a decrease in the average price charged by the other firms in the industry would mean that our firm will either have to lower its price or sell less. The properties of this demand function are discussed at length in Lovell [2000]. The demand function on Figure 2 was drawn with hypothetical parameter values $d_0 = 10$, $d_1 = 11$, $d_2 = 0.75$ and $n = 46$.

Demand function (7) implies that revenue will be

$$R(q_i) = d_1q_i + d_2\,\overline{p}_i\,q_i - q_i^2\,n^{1/2}/d_0. \qquad (8)$$

Therefore, profit will be

$$\pi(q) = R(q_i) - C(q) =$$

[3] Chamberlin[1950] called this the "large group" case; he also considered the more complicated "small group" case in which each firm considers the possible reaction of other firms in the industry in deciding what price it should set.

[4] It is customary in modeling monopolistic competition to assume constant marginal cost; i.e., to use a cost function involving the first two terms of (3). See, for example, Lovell [1970] and Dixit-Stiglitz[1977].

[5] For simplicity we invoke Chamberlin's symmetry assumption that all firms have the same cost and demand functions.

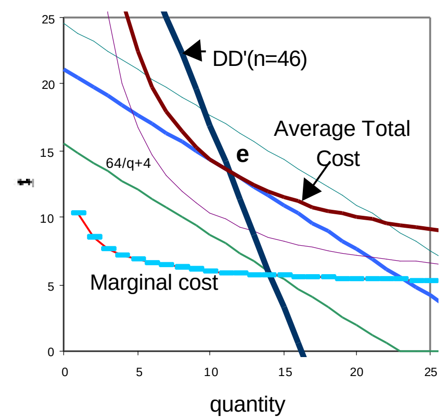
$$= - k_0 + (d_1 + d_2\,\bar{p}_i - k_1)q_i - q^2 n^{1/2}/d_0 - \alpha q_i^{1/2}. \tag{9}$$

As a necessary condition for profit maximization we must have

$$d\pi/dq_i = d_1 + d_2\,\bar{p}_i - k_1 - 2\,n^{1/2}\,d_0\,q_i - \alpha q_i^{-1/2}/2 = 0 \tag{10}$$

Long-run monopolistic competitive equilibrium also requires that $p = \bar{p}_i$ and that n adjusts so that $\pi = 0$. The details of the OLS-Monopolistic Competitive equilibrium are plotted on Figure 4 for the specified values of the parameters. At equilibrium point e we have $p = \bar{p}$ = \$13.5, q = 11.3, n = 46 and π = 0. This is a Nash equilibrium because it does not pay for any firm to change its price, given what the other firms are charging. It is also a long-run equilibrium because the entry of another firm would lead to negative profits.

Figure 4: OLS - MC Equilibrium

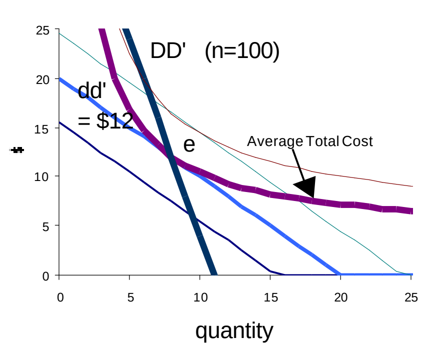


4. Just in Time

Just in Time (JIT), a managerial concept originating in Japan, is an alternative to batch processing.[6] The JIT strategy is to re-engineer the production process, and perhaps also modify the design of the product, so as to eliminate or to reduce drastically the set-up-cost.

At an ISIR conference at Wesleyan in 1987, Professor Hajime Yamashina of Kyoto University illustrated what is involved with a dramatic example. Toyota designed an automated factory for making the wide variety of gauges required for its autos. With batch processing substantial setup costs were incurred because of the idled labor and the loss of production during down time while the machines were adjusted to sw
gauge. In order to implement JIT, the rob
producing one type of gauge to another.
signal gauge down the assembly line, which
to the dummy gauge as it progressed to its s
read the dummy gauge's instructions to self-
be produced. Thus, the setup cost is reduce
With negligible setup cost the optimal inve
from equation (2).

Figure 5: JIT - MC Equilibrium



A variety of benefits accrue to the firm
addition to reducing inventories and their ca
trol. Inventories hide defects! With batch p
the production of an entire lot of defective p
is likely to be compounded because an assem
tempted to throw it back into the inventory
problem. Not only does this adverse selection mean that the next shift may inherit an inventory

[6] See also the online OR notes of J.E. Beasley, http://www.ms.ic.ac.uk/jeb/or/jit.html

with a high number of defects. It also means that a substantial number of defective parts will have been produced before the defect is properly recognized and corrective actions taken. That is why reducing inventory with JIT can make a key contribution to improved quality control.[7]

How will a monopolistically competitive industry respond to the adoption of just-in time? Clearly the innovating firm will reap higher profits, but how will price and the number of firms be affected if other firms also make the switch? Will consumers benefit from the switch to JIT?

The predictions that the theory of monopolistic competition makes about the effects of shifting to JIT management are illustrated on Figure 5.[8] The elimination of set-up and inventory carrying costs reduces the average cost function for the innovating firm to the new Average Total Cost curve. With its reduced cost structure, the best price for the JIT innovating firm to charge, given that the other firms in the industry continue to charge $\overline{p}$ = \$13.50, is

$$p_i = (d_1 + d_2 \overline{p}_i + k_1)/2 = \$12.56. \tag{11}$$

The corresponding quantity demanded, given that there are n = 46 firms in the industry, is

$$q_i = (d_1 + d_2 \overline{p}_i - k_1)d_0/2n^{1/2} = 12.6. \tag{12}$$

The innovating firm will enjoy profits of (12.56 – 4)12.6 – 64 = \$43.8, less the annual amortized cost of the investment required to institute JIT.

When other firms in the industry adopt JIT, their lower cost structure will intensify market pressures. The new Nash equilibrium, after all the firms have adopted JIT, given that n = 46, is

$$p_i^N = (d_1 + k_1)/(2-d_2) = \$12 \text{ for all i}, \tag{13}$$

with quantity

$$q_i^N = d_0[d_1 - (1-d_2)k_1)]/(c(n)(2-d_2) = 11.8. \tag{14}$$

Profits per firm are now

$$\pi_i^N = q^N(p^N-k_1) - k_0 = \$30.36. \tag{15}$$

Thanks to the copycats, the innovating firm now enjoys lower profits than anticipated, but customers benefit from consuming more of the product at lower prices.

This is unlikely to be the end of the story. Positive profits will attract more firms into the industry. Entry will continue until economic profit is driven down to zero. For this numerical example, the long run equilibrium predicted by the theory of monopolistic competition involves a price of \$12 but the number of firms increases to

$$n = \{d_0[d_1 - (1-d_2)\ \overline{p}\,](\overline{p}-k_1)/\ k_0\}^{1/(1-\gamma)} = 100. \tag{16}$$

Output for each firm is

$$q(\overline{p}\ ,n) = k_0/(\overline{p}-k_1) = d_0[d_1 - (1-d_2)\ \overline{p}\,]/c(n) = 8. \tag{17}$$

[7] In Lovell [1993] I show that the economy-wide adoption of JIT might contribute to economic stability within the context of a multi-sector model, but the role of price complications was excluded from the analysis.
[8] The graph and following equation are from Lovell [2000].

5. Summary and Conclusion

This paper draws upon the disciplines of Operations Research/Management Science and economics in demonstrating that the introduction of JIT may result in both lower prices and increased product variety. There may be added complications to the story. On the one hand, the analysis presented here neglected the possible effects of the adoption of just in time on fixed costs k_0. This coefficient may go up if it has to absorb the capital costs incurred in adopting the JIT technology. On the other hand, the adjustment may involve the adoption of multiple product lines by existing firms as well as the entry of new firms. If so, the fixed cost k_0 may be spread over several products and additional economies to scale may be realized. Never the less, at the end of the day both the innovating firm and its imitators are likely to find that the gains from improved managerial practice are far from permanent, but customers will benefit both from lower prices and from an increase in product variety.

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