PRODUCT DIFFERENTIATION AND MARKET STRUCTURE

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Product differentiation offers the consumer the opportunity to select commodities closely tailored to his individual preferences. The greater the variety of products offered, the more likely it is that a consumer will find available a commodity with the attributes he finds particularly desirable. But product differentiation generally has certain costs; for example, to change shape or color may involve setup costs in adjusting machinery or in the production of special dyes. Again, if differentiation takes the form of offering a product at a variety of locations, obvious costs are involved in duplicating stock and so forth at each store.

The effect of market structure on the degree of product differentiation is examined in this paper within the context of a model related to that used by Hotelling in his classic contribution to location theory [5]; the quantity purchased by each customer depends upon how closely the product is tailored to his individual tastes as well as upon price. After specifying the details of this environment in Section I, we explore how price and the degree of product differentiation depend upon market structure. Sections II and III contrast the behavior of an isolated monopolist with the way in which a franchise holder determines the optimal quantity to market in his assigned area. A liquor retailer whose marketing area is subject to zoning restrictions will behave in essentially the same way as the franchise holder. Section IV explains how the franchise lessor determines the optimal frequency of retail outlets. Equivalent problems involve the determination of the optimal density of stores in a retail chain and the extent of product differentiation that will maximize a monopolist’s profit.

We find in Section V that a “Colonel Sanders” interested in maximizing profits from the sale of chicken franchises may offer a quite different degree of product differentiation from that which would be established under conditions of free entry when all firms, existing and potential, regard the price charged by others as rigid in accordance with Bertrand’s Postulate. We also show how product diversity and price will be affected, even with free entry, if existing firms recognize the danger of entry and are careful to charge the “maximum entry-preventing price” rather than act under the erroneous assumption that they have to worry only about

*A preliminary draft of this paper was presented at the December, 1969, meetings of the Econometric Society. I am indebted to the Graduate School of Industrial Administration of Carnegie-Mellon University, to Wesleyan University, and to the National Science Foundation for released time and financial support. Although thanks are due to F. Trerey Dolbear, Jr., Charles Eckert, and Eugene Silberberg for helpful comments, I retain full responsibility for any remaining errors or heresies.

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competition from existing firms. Further, a quite different density of product types may be offered if each existing firm adjusts its price under the assumption that other firms already in the industry will follow it down by however much is necessary to retain their present customers.

Product differentiation raises a number of policy issues. For example, L. Preston considers customer territorial restrictions arising out of exclusive franchise arrangements in his analysis of Congressional investigations of restrictive distributional arrangements [10]. Again, J. Simon [14] found that the task of comparing the economic efficiency of state versus private retail liquor distribution is confounded by variations in the density of retail outlets. In Section VI we shall determine the price and degree of product diversity that serve to jointly maximize total welfare; this problem arises as a practical matter in states where liquor distribution is socialized. In Section VII we consider the optimal density of retail outlets, given that each individual store maximizes profits. This is the problem confronting a zoning commission charged with regulating entry when each retail outlet is operated as a private profit-maximizing enterprise. Our analysis shows that when variations in consumer taste give rise to product differentiation, market failure may take the form of an inappropriate variety of products offered at the wrong price.

1. The Environment

Hotelling considered a unidimensional market in his pioneering contribution to the theory of spatial competition. Customers are assumed to be uniformly distributed along a line, which may be interpreted as Main Street. The delivered price incurred by a particular customer is the sum of the price charged at the store plus transportation cost, which is proportional to distance shipped. Each customer buys from whichever supplier offers the lowest delivered price. As Hotelling emphasized, this model is capable of a variety of alternative interpretations involving forms of product differentiation other than location. Thus the “street” may be reinterpreted as representing a spectrum of possible colors, and “distance” the gap between the color regarded as ideal by the customer and the actual color offered by the supplier. Or the line may be interpreted as a measure of the degree of sweetness or sourness of cider, the position of the Democratic and Republican parties on the political spectrum, or even the position of religious denominations on the continuum from fundamentalism to free-thinking.

Hotelling’s Main Street may well constitute the simplest procedure for introducing product differentiation into the market for identical commodities encountered in the theory of competition. While even Babitt recognized that Main Street is a rather restrictive environment, it is
sufficiently rich to enable us to explore how the degree of product differentiation may be influenced by market structure. Let us proceed to spell out the nature of the assumptions involved.

*The Price Saw.* Suppose our retail outlet is located at point "O" on Main Street. If \( \sigma \) denotes the cost of shipping a unit of output one unit in distance, the price of the commodity delivered to a point located distance \( x \) from our store will be

\[
p_x = p + \sigma x
\]

(1)

The "price funnel," centered above the origin of Figure 1 indicates how

*Figure 1.*

the delivered price of our output increases for customers located at various distances from our store. Similar price funnels are drawn for neighboring retail outlets with dotted lines. The minimum price at which a customer at any location can obtain the commodity is indicated by the "price saw" formed by the lower envelope of the price funnels. Consequently, our market reaches to \( m_r \) on the right and \( m_l \) on the left; the total breadth of the market for our product is denoted by \( m \). Since \( m \) must be of such magnitude as to equate our delivered price with that of our competitor on the right, we must have \( p + m_r \sigma = p_r + (\mu_r - m_r) \sigma \). Hence \( m_r = (p_r - p)/2\sigma + \mu_r/2 \). An analogous expression holds for the left-hand limit of our marketing area, and the total breadth of our market is \( m = (p_l + p_r - 2p)/2\sigma + (\mu_l + \mu_r)/2 \). When the market happens to be symmetric, in the sense that \( p_l = p_r \) and \( \mu_l = \mu_r \), we have

\[
m = (p_r - p)/\sigma + \mu_r
\]

(2)

*Market Demand.* Suppose that all customers have the same demand curve

\[
q(p,x) = \max \{a - \beta(p + \alpha x), 0\}
\]

(3)

Demand curves of this form may arise when products are differentiated by some physical characteristic or when products are identical except for
location. For example, the demand curve will be of form (3) if utility function \( U = (a/\beta - ax)q - q^2/2\beta + y \) is maximized subject to the budget constraint \( Y = pq + y \), provided that \( Y > pq \). Here \( y \) denotes the consumption of other goods conveniently measured in units costing precisely one dollar, and \( Y \) is the money income of the consumer. The parameter \( x \) may be regarded as measuring the gap between the commodity's actual characteristic and the characteristic regarded as ideal by the consumer; e.g., \( x = |m - m^o| \), where \( m \) is the actual color, appropriately indexed, and \( m^o \) the optimal color. The uniformity assumption alluded to earlier implies that consumers are evenly distributed in terms of preferences along the entire color spectrum.\(^1\) Note that demand curve (3) also arises if the utility function takes the form \( U = aq/\beta - q^2/2\beta + y \), while the budget constraint is \( Y = (p + ox)q + y \), where \( x \) now denotes distance and \( o \) transportation cost.\(^2\)

Unless \( \beta = 0 \), a special case with which Hotelling was primarily preoccupied,\(^3\) the furthest conceivable point at which we can sell our product, even in the absence of competitors, is that point at which the sum of price plus shipping costs equals \( a/\beta \); this maximum marketing distance is

\[
x^* = (a - \beta p)/a\beta
\]

Thus, the effective range of our marketing activities to the right of our retail outlet is \( m_r^* = \min (m_r, x^*) \); similarly, our marketing distance to the left is \( m_l^* = \min (m_l, x^*) \); the total effective breadth of our market is \( m^* = m_l^* + m_r^* \). Total sales of our product are now

\[
Q(p, m_l^*, m_r^*) = \int_{m_l^*}^{m_r^*} (a - \beta p - \beta om)f(m)dm
\]

where \( f(m) \) denotes the density of customers at any point \( m \) along Main

1. To avoid complexity we retain Hotelling's assumption that the market is unidimensional. Kuehn and Weiss [7] discuss the "sudiness" characteristic of soap in precisely these terms, but they suggest that the distribution is bimodal rather than uniform. Of course, the form of the distribution is sensitive to whatever scaling convention is employed; our analysis is restricted to the case in which the distribution is uniform when distance is scaled so as to be proportional to utility loss (or transportation cost).

2. For much of the argument of this paper it is not necessary to assume that demand is a linear function of delivered price. It does prove essential, in talking about nonspatial product differentiation, to suppose that the parameter \( x \) enters into the utility function in such a way that quantity purchased by each consumer is a function of \( p + ax \). Further, when the time comes to discuss certain welfare issues, it proves useful to be able to work in terms of consumers' surplus, a concept that becomes ambiguous if the marginal utility of money is not assumed constant. Given the restricted nature of these other assumptions, the additional hypothesis of linearity is relatively painless; and it is a particular convenience in considering possible empirical applications.

3. Hotelling [5, pp.183-84] did sketch the way in which his argument had to be modified to take into account the case of less than completely inelastic demand. The analysis was elaborated upon by Smithies [15, pp.423-29]. Much of the present paper constitutes a similar generalization of arguments of Tobin [17] and Vickrey [18, pp.334-36], both of whom considered free entry in the case where \( \beta = 0 \); Vickrey avoided end-point problems by having Main Street form a loop while Tobin conceived of Main Street as being infinitely long.
Street. Let us suppose, following Hotelling, that the customers are uniformly distributed; \( f(m) = f(m') \) for all \( m \) and \( m' \). Normalizing, so that \( f(m) = 1 \), we obtain

\[
Q(p, m^*_l, m^*_r) = (a - \beta p) m^*_l - \beta \sigma ((m^*_l)^2 + (m^*_r)^2)/2
\]

It is easily verified that, given the price charged by the two adjacent neighbors, maximum revenue will be obtained if the store's location is such that \( m^*_l = m^*_r \). Let us take advantage of the algebraic simplification permitted by assuming that the store's location is optimized in this way; we have for total sales

\[
Q(p, m^*_l) = (a - \beta p) m^*_l - \beta \sigma (m^*_l)^2/4
\]

With market symmetry, average sales per mile are

\[
q_s(p, m^*_l) = Q/m^*_l = a - \beta p - \beta \sigma m^*_l/4
\]

Of course, the domain of definition of this function is restricted to nonnegative \( p \) and values of \( m^*_l \) satisfying

\[
m^*_l \leq 2x^* = 2(a - \beta p)/\beta \sigma
\]

This inequality recognizes that the market may not be completely saturated in the sense that some customers elect not to purchase any of the commodity because they are located so far from the nearest store that the delivered price is above \( a/\beta \). Since (7) implies \( \beta \sigma m^*_l/2 \leq a - \beta p \), we find on substituting into (6) that

\[
q_s \geq \beta \sigma m^*_l/4
\]

is the market saturation boundary. We summarize the market conditions as follows

\[
q_s(p, m) = \begin{cases} 
  a - \beta p - \beta \sigma m/4 & \text{if } m < 2(a - \beta p)/\beta \sigma \\
  (a - \beta p)/2 & \text{otherwise}
\end{cases}
\]

for \( 0 < p < a/\beta \) and \( m > 0 \).

Iso-price lines indicating the response of sales to changes in \( m \), given \( p \), are plotted on Figure 2; all have slope \(-\beta \sigma/4\). The highest iso-price line, \( q_s(0, m) \), indicates how average sales respond to changes in \( m \) when the good is given away by the retail outlet; for positive price, lower iso-price lines are generated; in particular, iso-price line \( q_s(\delta, m) \) indicates the relationship between sales per mile and market breadth when the com-

\[\text{4. In the case in which the two neighboring competitors charge the same price, this amounts to locating one's own store midway between them.}\]
modity is sold at marginal cost. As price approaches $a/b$, the iso-price lines collapse to the origin. All the iso-price lines terminate at the positively sloped market saturation line in recognition of condition (8).

Cost Conditions. We proceed under the simplest possible assumptions in specifying cost conditions. Specifically, we assume that cost conditions are the same at each retail outlet and that the store's total cost of production is simply

$$C(Q) = \gamma + \delta Q$$

Here $\gamma$ denotes the store's fixed cost and $\delta$ marginal cost. Included in $\gamma$ are the carrying cost of minimum display inventory, licensing fees, rent, overhead personnel and other such costs that may be regarded, at least as a first approximation, as independent of volume. Marginal cost $\delta$ includes the wholesale cost of the product, the cost of packaging, etc. When product differentiation arises from color or stylistic factors, we can adopt a slightly different interpretation, for then $\gamma$ may be interpreted as having to do with the setup costs involved in preparing a paint of a particular color, adjusting the machine to produce an item of specified size or shape, and so forth.\(^5\)

II. THE ISOLATED MONOPOLIST

The isolated monopolist is in the fortunate position of having any potential competitors located far enough away to be priced entirely out of the market by transport costs. Obviously, an isolated monopolist operating on Hotelling's Main Street is not likely to remain alone for long. If he finds business profitable, he will be inclined to form a chain by opening additional retail outlets. And if limitations on the availability of entrepreneurial ability lead him to decline an opportunity for profitable expansion, other firms may be induced to enter the industry. A lazy monopolist protected by patent or trademark may find it profitable to license the sale of competing products even when this may tend to infringe to some extent upon his own marketing activities. But while isolated monopoly may not always constitute a stable form of market organization, it must

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\(^5\) With $a = 0$ but $\gamma > 0$ the problem degenerates into the case of single producer; but with $\gamma = 0$, there is likely to be an infinite number of product types spread uniformly along Main Street, a situation discussed by Kaldor [6]. To simplify the analysis we exclude the possibility of storing the commodity from one period to the next.
be studied before we can examine such phenomena as free entry and the operations of a franchise system.

The breadth of the market serviced by the isolated monopolist is bounded by that point at which his delivered price equals $a/\beta$, for no customer will purchase the commodity for more than that amount. Equation (7) reveals how the breadth of his market is influenced by price; demand at each point within this range is given by (3).\(^6\) Eliminating $m^*$ from (5), with (7), reveals

$$Q(p) = (a - \beta p)^2/\beta \sigma$$

Total profits are $\pi = (p - \delta)(a - \beta p)^2/\beta \sigma - \gamma$. Differentiating with respect to price reveals as a necessary condition for profit maximization $d\pi/dp = (a - \beta p)^2/\beta \sigma - 2\beta(a - \beta p)(p - \delta)/\beta \sigma = 0$ or

$$p = a/3\beta + 2\delta/3$$

Note that this optimal base price is independent of shipping costs, the parameter $\sigma$. Further, two-thirds of any increase in marginal cost will be passed on to the consumer in the form of higher prices. We then have from (11) that total sales are $Q_m = 4(a - \beta \delta)^2/9\beta \sigma$. Further, the breadth of the market is

$$m_m = 2x^* = 4(a - \beta \delta)/3\beta \sigma$$

Finally, average sales are

$$q_a = (a - \beta \delta)/3$$

Thus, the profit-maximizing monopolist will have average sales equal to 1/3 of what he would sell to customers zero distance from his store if he were to pursue a marginal cost pricing rule. Finally, we note that at best the isolated monopolist reaps a return in excess of variable cost of $\pi + \gamma = 4(a - \beta \delta)^2/27\beta \sigma^2$. In the short run an existing plant will be shut down if variable costs $\delta$ are so high that $a - \beta \delta = q(\delta, 0)$ if 0. In the long run the monopolist will fade out of business, at least in the absence of a subsidy, if fixed costs are too high; specifically, output will be zero in the long run unless

$$\gamma \leq 4(a - \beta \delta)^3/27\beta \sigma^2$$

How the isolated monopolist behaves may be visualized with the aid of Figure 3. The market saturation line has been borrowed from Figure 2; it emanates from the origin with slope $\beta om/4$. The isolated monopolist’s problem is to pick the best point on this line. The effect of charging

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\(^6\) See Beckmann [2, pp. 30-32] for a discussion of discriminatory pricing and a generalization to two dimensions.
marginal cost is given by point \( a \) where the market saturation line is crossed by iso-price line \( q_a(\delta, m) \), but this would involve a loss of \( \gamma \). Point \( M \) denotes the profit maximizing solution; as equation (14) requires, \( q_a \) is precisely one third of \( q_a(\delta, 0) \). We see that only two-thirds as many customers are served as at point \( a \), which involved marginal cost pricing. Further, sales per mile of \( q_a \) are exactly half as large as they would be over a market of width \( m_m \) with marginal cost pricing.

The effects on profits, price and market size of changes in the parameters of the model can be determined by partially differentiating the relevant equations of this section. The imposition of a licensing fee would increase \( \gamma \), and it is easily verified that this would have no effect on the behavior of the monopolist; he remains at point \( M \) on Figure 3, even in the long run, unless the tax raises \( \gamma \) to the point where condition (15) is violated. It is surprising to observe that any tax imposed upon transportation facilities, by changing \( \sigma \), will be borne entirely by the customer, for this parameter does not affect the optimal base price \( p \) of equation (12). Further, \( q_a \), average shipments over the firm's market, is unaffected. However, the width of the market is reduced when \( \sigma \) increases, and as a result total sales and profits fall proportionately. An increase in \( \delta \), the parameter denoting marginal cost, would result from the imposition of a specific sales tax; it is easily verified that the producer will absorb one third of the tax.

III. FRANCHISES, ZONING RESTRICTIONS, AND THE CUSTOMER RETENTION POSTULATE

Now consider the case in which the width of the market is prescribed to the firm. Subject to this restriction, the firm is free to optimize on price. This problem often confronts the lessor of a franchise—the "franchise." It also confronts a firm protected from encroachment into its market territory by zoning restrictions, and it may even arise when legalistic restrictions are absent. Specifically, the problem is relevant for a firm whose belligerent neighbors always adjust their price by whatever amount.

7. A franchise may be granted subject to restrictions on pricing policy as well as geographic coverage; however, this only constitutes benevolent guidance as to optimal behavior by the lessee. For there is no conflict of interest between the parties to the franchise agreement on pricing policy; of course, certain terms of a poorly drawn franchise may encourage the lessor to follow strategies that do not maximize combined profits.
is required to prevent encroachment on their market territory. Customer retention constitutes a variant on the concept that Modigliani [9, p.217] has christened "Sylos' Postulate." What all three of these situations have in common is that only the depth of the market is affected by price adjustments; that is to say, a price change may affect sales to existing customers, but it will not affect the breadth of the market.

Profit per mile is

\[
\pi_a = \frac{\pi}{m} = (p - \delta)q_a - \gamma/m
\]

Since \(m\) is fixed, maximizing profits per mile is equivalent to maximizing total profits, so we can focus upon \(\pi_a\). To suppress price from this equation we note from (6) that

\[
p = \frac{a}{\beta - \alpha m/4 - q_a/\beta}
\]

and also \(q_a(\delta, m)/\beta = a/\beta - \alpha m/4 - \delta\). Thus we have

\[
\pi_a = q_a(\delta,m)q_a/\beta - q_a^2/\beta - \gamma/m
\]

To find the iso-\(\pi_a\) curves constituting the loci of all points with average profits \(\pi_a\) we rewrite (18) as a quadratic in \(q_a\)

\[
q_a^2 - q_a(\delta,m)q_a + \delta(\pi_a + \gamma/m) = 0
\]

with roots

\[
q_a = q_a(\delta,m)/2 \pm 0.5 [q_a(\delta,m)^2 - 4\delta(\pi_a + \gamma/m)^{0.5}]
\]

Thus, the iso-\(\pi_a\) curves must be symmetric about the \(q_a(\delta,m)/2\) line already plotted on Figure 3. To verify that the \(q_a(\delta,m)/2\) line yields the profit maximizing value of \(q_a\) for given \(m\), we differentiate (18) partially with respect to \(q_a\), obtaining \(\partial\pi_a/\partial q_a = q_a(\delta, m)/\beta - 2q_a/\beta\) and \(\partial^2\pi_a/\partial q_a^2 = -2/\beta < 0\). So equating the first derivative to zero yields the maximizing average quantity

\[
q_{ma} = (1/2)q_a(\delta,m) = (a - \beta \delta)/2 - \beta \alpha m/8
\]

This can be read directly off Figure 3, given \(m\). The optimal quantity to market is simply

\[
Q_f = mq_f = mq_a(\delta,m)/2
\]

which can be sold, according to (17), at price

\[8. \text{The phenomenon considered by Modigliani [9] in his review of Sylos [16] involved a slightly less belligerent policy under which existing firms reduce their price only as much as required to continue to sell present output. Within the context of product differentiation, however, the customer retention postulate may be more interesting. Certainly, it constitutes a polar form of reaction to a competitive threat.}\]
(23) \[ p_f = a/2\beta + \delta/2 - \alpha m/8 \]

We find that

(24) \[ q_{fa} = (p_f - \delta)\beta = a/2 - \delta\beta/2 - \alpha m/8 = q_f(\delta, m)/2 \]

and so the maximum level of profits, for given \( m \), is:

\[ \pi_f = (p_f - \delta)Q_f - \gamma = m\beta(p_f - \delta)^2 - \gamma \]

(25) \[ = m\beta(a/2\beta - \delta/2 - \alpha m/8)^2 - \gamma \]

Of course, given \( m \), there obviously exists a ceiling on fixed costs that must not be exceeded if the retail outlet is to operate without loss.

How environmental changes influence the price charged by a franchise holder may be determined by differentiating (23). We note that \( \partial p/\partial \alpha = -m/8 \); thus, an increase in transportation costs (e.g., a transport tax) will cause a reduction in the price charged for the commodity at the store. Customers located close to the store will actually benefit from the transport tax, although more distant customers as well as the franchisee will suffer. Note also that \( \partial p/\partial m = -\alpha/8 \); an increase in the franchise area, which might be interpreted as a reduction in competitive pressure, serves to reduce the price charged by the franchise operator; again, customers so fortunate as to be located close to the store benefit.

What size franchise is optimal, from the point of view of the franchise lessor or in terms of general welfare, is an issue that will demand our attention in a moment, but it is obvious that the breadth of the market that will be serviced by an individual franchisee is bounded by \( m_m \) of equation (13). Furthermore, there exists a minimum market breadth, call it \( m_{min} \), that must be allocated to each franchisee in order that he may operate without loss. Only if fixed costs are so high that (15) holds as a strict equality will these two values coincide.

In an industry characterized by customer retention behavior rather than franchise restrictions, the range of store sizes must still fall between \( m_{min} \) and \( m_m \). However, less product variety may result. Suppose that the stores happen to be spaced just slightly less than \( m_{min} + m_m \) units apart, so that each monopolist services an area of width \( m_m \). Although there remains an unsatisfied fringe of customers who are priced completely out of the market, the existence of positive profits will not induce a new retailer to enter the market if he is convinced that existing firms will cut prices by whatever is required to retain their present customers.

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9. Obviously, \( m_{min} \) must be a root of the cubic equation obtained by equating the right-hand side of (25) to zero. Note, also, that as one moves westward from \( M \) along line \( q_{fa} \) on Figure 3 profits decline monotonically to \(-\gamma \); \( m_{min} \) corresponds to the intervening point where \( \pi = 0 \).
IV. OPTIMAL FRANCHISING—PROFIT PER MILE MAXIMIZATION

Now consider the problem from the point of view of a franchise lessor—Colonel Sanders himself. If he grants an exclusive franchise to a market of width \( m \), \( m_{min} < m < m_{max} \), the franchisee will market \( Q_f \) at price \( p_f \), as prescribed by equations (22) and (23). But what is the optimal size territory \( m \) for the Colonel to give to each franchisee? In practice, this is a difficult problem, for the assumption that customers are uniformly distributed along a line is no more than a convenient abstraction. Even with this uniformity assumption, the problem would be difficult if we were to follow such writers as Hotelling in supposing that the line is of finite length, for it would not necessarily be true that the profit-maximizing strategy involves either markets of the same size for all stores or uniform pricing. The problem is subject to considerable simplification if we may be permitted to assume that Main Street is sufficiently long to permit us to neglect “end-point” complications. The assumption that Main Street is infinite in length also serves to eliminate from consideration one form of market failure that has already been subjected to careful scrutiny. It is well-known that the market solution to the finite road problem may not be socially optimal; however, Chamberlin [3, Appendix C] demonstrated that the loss falls as the number of retail outlets increases, at least when demand is insensitive to price. We shall show that once it is admitted that demand is sensitive to price another form of market failure may be encountered which is not the consequence of end-point distortions.

Colonel Sanders’ total profits are likely to be infinite under a wide variety of strategies if Main Street is infinitely long; we may circumvent this conundrum by assuming that he desires to maximize profits per mile, \( \pi_a = \pi/m \).\(^{10}\) To find the optimal market breadth for Colonel Sanders to grant to each franchisee we differentiate (18) partially with respect to \( m \), obtaining

\[
\frac{\partial \pi_a}{\partial m} = \left( \frac{q_a}{\beta} \right) \left[ \frac{\partial q_a}{\partial m} \right] + \gamma/m^2 = -\alpha q_a/4 + \gamma/m^2
\]

Since \( \beta^2 \pi_a/\partial m^2 = -2\gamma/m^3 < 0 \), equating equation (26) to zero yields the optimal market breadth for given \( q_a \)

\[
(27) \quad m = \left( 4\gamma/q_a \right)^{1/3}
\]

\(^{10}\) The resulting solution is a useful approximation when the road is long but finite. Maximizing profits per mile is analogous to a strategy frequently employed in operations research: when asked how to behave when the discount rate is to be neglected and the planning horizon is to be regarded as infinite, it is often useful to consider the task of maximizing average profit per period. A limitation of this strategy is that one can obtain the same average profits even if one procrastinates indefinitely in the adoption of the strategy that maximizes profits per period.
This curve is plotted on Figure 4. Given \( q_s \), this curve reveals the optimal market size; obviously, iso-\( n_a \) curves revealing the loci of all points yielding a specified profit per mile must have zero slope where they cross this line. Now it will be remembered that for given market size \( m \) the individual franchise holder will maximize his profits by selling half as much as he would sell over his assigned market if he priced at marginal cost; this is indicated by the \( q_s(\delta, m)/2 \) line, reproduced from Figure 3. Equation (23) again dictates the optimal price.

At point \( F \), profits per mile are maximized with respect to both \( q_s \) and \( m \), and implicitly with respect to price. The franchise system serves to maximize profits per mile, but how these profits are divided between Colonel Sanders and the individual operators depends upon the terms of the contract. That the franchise system yields the same type of behavior as is displayed by a chain store maximizing profit per mile is not too surprising.

As indicated by the arrows on the graph, in only a few steps the system will converge close to the optimum point \( F \) even if the franchise granted each contract period is based on the preceding period’s average sales. Of course, relocation at the expiration of each contract period may be unduly expensive if the retail outlet involves highly specialized facilities. In theory a portable retail outlet is advantageous if an iterative solution is to be relied upon, but in practice relocation may be as hard as converting to the metric system.

Let us explore the effects of a lump-sum tax placed upon each retail outlet, possibly in the form of a licensing fee. Obviously, a sufficiently high tax would make the business totally unprofitable, for the tax raises the fixed cost \( \gamma \) of operating a retail outlet. Remember that such a tax will have no effect on the behavior of the isolated monopolist unless it is so high as to cause condition (15) to be violated; i.e., point \( M \) is not affected by the tax. How will the tax affect the franchise operation? It will not change the \( q_s(\delta, m)/2 \) line on Figure 4, for \( \gamma \) does not enter into equation (6). But a licensing fee will cause the curve defined by equation (27) to shift to the right, and point \( F \) slides along \( q_s(\delta, m)/2 \) in a southeastward direction toward \( M \). As a consequence average sales per mile \( q_s \) must fall. Furthermore, the optimal distance between retail outlets
is larger; the size of the retail franchises offered by an optimizing Colonel Sanders is increased. Or to put it another way, the effect of the licensing fee is to reduce the degree of product variation offered in the market place.

Although the effect of the license fee is inconvenience for the customer, in the sense that retail outlets are further apart, it is by no means obvious that customers are worse off. The larger \( m \) implies that the optimal price is lower. That the holder of a larger franchise will charge a lower price is clear from equation (23); it is also obvious from the graph that since the iso-price lines have the same slope as \( q_a(\delta, m) \), they must be steeper than \( q_a(\delta, m)/2 \). Clearly, the lower price is beneficial for the customer located close to the retail outlet. But those located far away lose. Whether or not the net effect of less variety and lower price is beneficial in the sense that the gainers can compensate the losers is a question we shall consider in a later section.

V. BERTRAND'S POSTULATE

What should our firm do in order to maximize profits under the naive assumption that competitive neighbors will maintain their price regardless of the marketing strategy we decide to follow? In contrast to customer retention behavior, there now exists the hope that customers can be attracted away from competitors by lowering price. But along with the possibility of market penetration, there is also the danger that a price increase may cause customers to be lost to competitors. This form of conjectural variation underlies Bertrand's theory of oligopoly. After investigating the behavior of the representative firm, we shall consider the market implications of free entry. Again we assume market symmetry.

As a first step toward determining the optimal strategy for the firm, let us define

\[
(28) \quad w = p_r + \mu a/2
\]

which is the delivered price that the product of our competitor will sell for at the site of our store. Then from (2) we have as the relationship under symmetry between our market breadth and our price

\[
(29) \quad p = w - \sigma m
\]

Furthermore, we find on substituting into (6) that our average sales will be

\[
(30) \quad q_a = a - \beta w - 3\sigma m/4
\]

Our firm wishes to maximize total profits

\[
(31) \quad \pi = (p - \delta)mq_a - \gamma
\]

\[
= (w - \delta - \sigma m)(a - \beta w + 3\beta m/4) - \gamma
\]
This function is a cubic in \( m \), and we note that \( \pi = -\gamma \) if \( m = 0 \), if \( p = \delta \), or if \( q_a = 0 \). We must find the profit maximizing value of \( m \), subject to the obvious constraint that it must be nonnegative. Differentiating (31) with respect to \( m \) yields

\[
\frac{\partial \pi}{\partial m} = -\frac{9}{4} \beta \alpha ^2 \left\{ m^2 + \left[ \frac{8}{9} \left( \frac{a - \beta w}{\beta \alpha} \right) - \frac{6}{9} \left( w - \delta \right) \right] m - \frac{4}{9} \left( \frac{a - \beta w}{\beta \alpha} \right) \left( \frac{w - \delta}{\beta \alpha} \right) \right\} = 0
\]

Clearly, a necessary condition for a maximum is that \( m \) be a root of the quadratic inside the braces. Inspection of (31) reveals that it is the largest root that is relevant, and we have

\[
m = \frac{3}{9} \left( \frac{w - \delta}{\alpha} \right) - \frac{4}{9} \left( \frac{a - \beta w}{\beta \alpha} \right)
+ \left\{ \left[ \frac{3}{9} \left( \frac{w - \delta}{\alpha} \right) - \frac{4}{9} \left( \frac{a - \beta w}{\beta \alpha} \right) \right]^2 + \frac{4}{9} \left( \frac{a - \beta w}{\beta \alpha} \right) \left( \frac{w - \delta}{\beta \alpha} \right) \right\}^{\frac{1}{2}}
\]

This expression may be substituted into equations (29) and (30) to determine optimal price and sales volume.

Additional insight into the problem may be obtained by considering Figure 5. The positively sloping line labeled \( q_a(w, m) \) is equation (30), and indicates the quantity that the firm can sell as a function of \( m \), given the pricing policy of competitors as summarized by the value of \( w \); or to put it another way, it reveals average sales, given price as prescribed by equation (29). The firm’s problem is to find the point on this line that yields maximum profits. Several iso-profit curves have been plotted on the graph, and tangency point \( B \) indicates the profit maximizing breadth of market. If the price of our competitor’s product delivered to our location changes from \( w \) to \( w' \), we will have a new line \( q_a(w', m) \) summarizing the opportunities available to the firm, and tangency point \( B' \) will constitute the profit-maximizing solution. The negatively sloping curve passing through points \( B \) and \( B' \) denotes the set of conceivable tangency solutions obtained by varying the value of \( w \). Of course, the retail operation is viable when \( w \) is large enough for the tangency solution to yield positive profit.

Since the iso-\( \pi \) curves will prove of considerable interest in the analysis
that follows, certain of their properties warrant detailed attention. First of all, if we multiply (16) by \( m \) we obtain

\[
\pi = m \pi_a = q_a(\delta, m)q_a m/\beta - q_a^2 m/\beta - \gamma
\]

and by a procedure analogous to that used in deriving (20) we have

\[
q_a = q_a(\delta, m)/2 \pm \left[ q_a(\delta, m)^2 - 4(\pi + \gamma) \right]^{1/2}/2.
\]

Consequently, the iso-\( \pi \) curves, like iso-\( \pi_a \) curves, must be symmetric about the line \( q_a(\delta, m)/2 \). Differentiating with respect to \( q_a \) indicates that the optimal quantity to market for given \( m \) is revealed by equation (21); thus the iso-\( \pi \) curves have infinite slope only at points where they cross the \( q_a(\delta, m)/2 \) line. Differentiation of (33) with respect to \( m \) reveals

\[
\frac{\partial \pi}{\partial m} = \pi_a + m \left( \frac{\partial \pi_a}{\partial m} \right)
\]

\[
= \int \left( q_a(\delta, m)/\partial m \right) q_a(\delta, m)/\beta + q_a(\delta, m)q_a/\beta - q_a^2/\beta
\]

Hence, iso-\( \pi \) curves have zero slope where they cross the line

\[
q_a = a - \beta \delta - \beta \sigma m/2
\]

This line is plotted on Figure 5. Note that both the iso-\( \pi \) and iso-\( \pi_a \) curves have zero slope at point \( x \); from (34) it is clear that this implies that profits must be zero at this point.

On Figure 5 we observe that the \( q_a(w', m) \) line is just tangent to this zero iso-\( \pi \) curve. Consequently, point \( B' \) denotes a situation in which firms are located so densely that at best only zero profits can be achieved. Although firms might still make positive profits even if spaced closer than the distance \( m_n' \), this would involve a more judicious pricing practice—e.g., customer retention behavior. Note also that point \( B' \) is further west than \( x \), and \( x \) in turn is west of \( F \). This means that if the ultimate equilibrium under the Bertrand Postulate turns out to be characterized by zero profits (point \( B' \)), stores will be located more densely than under an optimal franchise scheme.11

This argument requires a minor modification if it is conceivable that a store may price its immediate neighbors completely out of the market. That is, at the point where \( p = p_r - \sigma \mu/2 \), the price of the commodity delivered at the next store will equal the neighbor's price. Any further reduction in price will capture all of the competitor's market. At this point the next pair of adjacent firms—one's competitors once removed so to speak—become relevant in deciding upon appropriate pricing policy. The analysis proceeds much as before. Let \( w_2 \) be the delivered price of the output sold by the next pair of competitive firms. On Figure 6 we have

11. If fixed costs are so high that \( F \) coincides with \( M \), then points \( B \) and \( x \) will also coincide, and all forms of viable market behavior are identical.
plotted curve \( q_2(w, m) \). The iso-price line for price \( p_r - \sigma \mu/2 \) is also indicated. The opportunities available to the firm under the Bertrand Postulate are indicated by the heavy jagged line on the graph. Of course, it seems excessively naive for a firm to believe that its competitors will not modify their pricing strategy when priced completely out of the market.

Let us now focus on the issue of entry. The concept of “maximum entry-forestalling price” has been discussed at length in the literature.\(^\text{12}\) Within the context of our problem, the question of whether entry will be forestalled or not hinges in part upon the expectations of potential entrants about the behavior of existing firms. Under the assumption that potential entrants regard the price charged by existing firms as rigid, the conditions of entry can be specified in terms of the variables \( m \) and \( q_a \); the best price for the entrant to charge is then given by (6); we shall use the subscript \( ef \) to indicate entry-forestalling values of these variables.

As a first step toward determining the conditions under which entry will be forestalled, let us return to Figure 1. A potential entrant considering location \( 0 \) midway between two existing firms both charging price \( p_r = p \), will be confronted with competition whose intensity is summarized by \( w \), the price at which the competing merchandise sells at the site of the proposed store. Entry will be forestalled if \( w \) is less than the value \( w' \) corresponding to zero profits on Figure 5.\(^\text{13}\) That is to say, existing firms will forestall entry if

\[
(36) \quad w = p_r + \mu \sigma/2 \leq w'
\]

Without entry the existing store services a market of width \( m = \mu \). Thus, the maximum entry-forestalling price as a function of \( m \) is \( p_{ef} = w' - \mu \sigma/2 \). Substituting into (6) reveals the minimum entry-forestalling average quantity is \( q_{ef} = a - \beta w' + \beta \sigma m/4 \). This line is plotted on Figure 7.

Note that there exists a range of indeterminateness in the situation in which all firms, existing and potential, regard their rival’s price as fixed. That is to say, if stores are operating anywhere along the expansion curve

\(^{12}\) For a textbook treatment, see Bain [1, p. 272].

\(^{13}\) We restrict our attention to the case in which the potential entrant does not contemplate pricing the existing firms completely out of the market.
positive profits are being realized and yet entry is not profitable. We have an indeterminate situation of neutral equilibrium.\(^{14}\)

Positive profits may not suffice to attract entrants, even though potential entrants would enjoy precisely the same cost functions as existing stores. Profits can persist because potential entrants recognize that there is insufficient space to squeeze between existing firms, given their present price and location.\(^ {15}\)

It is easily shown that profits equal to fixed costs will definitely suffice to make entry worthwhile. Since an entrant who charges the same price as existing firms will achieve half their market breadth, he will sell at least half as much as existing firms, and more unless \(\beta = 0\). But half the sales of the existing store will yield enough revenue to cover fixed costs, if the existing stores have revenue exceeding variable cost by twice the magnitude of fixed costs; i.e., \(\pi \geq \gamma\). Even when profits are less than fixed costs, entry may still prove worthwhile. Not only will \(\beta > 0\) mean that the necessary sales volume may be realized; it may also be possible to do better by not charging the same price as the existing firms. Under the Bertrand Postulate, fixed costs thus constitute an upper bound on the profits that existing firms can realize without inviting entry.

Let us now consider a variant on the Bertrand Postulate: suppose that a franchise lessor (or chain-store management) is concerned about the entry of potential rivals. Suppose that the franchise management assumes that the potential entrants will have the same cost function as its own retail outlets and, furthermore, that the entrants will regard the franchise's price as invariant with respect to the price charged by the entrant. Under these circumstances it is essential for the franchise operator to determine whether its marketing strategy requires modification in order to forestall entry. Suppose as suggested on Figure 7, that the line \(q_{ef}\) is

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14. The existence of this indeterminancy in no way contradicts Chamberlin's *Theory of Monopolistic Competition* [3]. Admittedly, only at zero-profit point \(B'\) is it possible to characterize the firm's equilibrium in terms of the tangency of its demand and average cost curves; elsewhere on \(B'-B\) the existence of positive profits means that the demand curve lies above the average cost curve. But Chamberlin argued (p.113) that the forces of monopolistic competition would not always yield the tangency solution, and that profits might remain scattered throughout the system.

15. Whether it is reasonable for potential entrants to exclude the possibility that existing firms would move over in the face of entry may be an issue open to debate. But it is clear that when all potential entrants make this assumption, whether erroneously or not, subsequent developments will not contradict it.
above point $F$. In this situation it behooves the franchise lessor to locate his retail outlets more densely and/or to lower prices; graphically, the solution to the constrained maximum problem involves selecting that point on the $q_{df}$ line that is tangent to the highest iso-$\tau_d$ curve. This result illustrates the well-known proposition that even the threat of entry may serve to benefit the consuming public. On the other hand, firms already in the market may attempt to discourage potential entrants by indicating that they will respond with aggressive price cuts rather than behave in passive conformity to the Bertrand Postulate.

VI. WELFARE CONSIDERATIONS

What is the socially optimal degree of product differentiation? This issue arises as a practical matter in the setting of zoning regulations and in the imposition of licensing fees on retail establishments. It also arises in the operation of socialized industries; for example, it confronts the liquor authority in states where liquor is distributed in state-owned stores. The question does not admit a simple answer, for we have seen that increased product differentiation may involve higher prices. Difficult interpersonal comparisons may arise when nearby and distant customers experience differential effects.

One possible objective might be to maximize the degree of product differentiation, subject to the requirement that no retail outlets suffer financial loss. According to Beckmann [2, p. 44], this criterion was advocated by Losch. From Figure 5 we can observe that point $L$ satisfies this criterion. Point $L$ involves more product differentiation than will be offered by a profit-maximizing franchise operation. While this solution might be obtained if individual firms engage in customer retention behavior, it is not attainable under the Bertrand Postulate, even with free entry. Preston [10, p. 517] has suggested that point $L$ serves to maximize consumer welfare, arguing that "there is every reason to believe that the customer population is not directly harmed, and quite possibly benefited, by the establishment of a maximum number of viable distributorships."

Although Preston fails to note that the higher price involved in maximizing the number of viable outlets means that some customers must surely lose, he does recognize that it is conceivable that the establishment of the maximum number of viable distributors may result in an overcommitment of resources to distribution.

Let us use the concept of consumers' surplus in evaluating various forms of market organization. To determine the total consumers' surplus gener-

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16. In an interesting comparison of liquor prices in states where liquor is distributed by private retail outlets with prices in states where liquor distribution is a government enterprise, Julian Simon [14] found that socialized distribution appeared to be more efficient; but it was not possible to evaluate either the density of retail outlets or the variety of products offered in each store.
ated by a store's activities, we must first find the consumers' surplus as a particular point on Main Street and then integrate over the entire breadth of the market served by the store. If the commodity is delivered at price \( p_x \) to customers at a particular address on Main Street, they will purchase the quantity prescribed by demand function (3); because of the simple form we have assumed for the demand curve, the area of the familiar triangle representing the surplus enjoyed by the consumer is simply \( s(p_x) = (a/\beta - p_x)(a - \beta p_x)/2 \), provided that \( p_x < a/\beta \). Substituting from (1) reveals that customers located at distance \( x \) from a store charging price \( p \) will enjoy a surplus of \( s(p, x) = \alpha^2/2\beta - a(p + ax) + \beta(p + ax)^2/2 \). If the State Store serves a market of width \( m \), total consumer surplus generated by its activities will be:

\[
(37) \quad S(p, m) = \int_{-m/2}^{m/2} s(p, x) \, dx = \frac{1}{2} \alpha^2 \beta m/2 - (a - \beta p)m^2/4 + \beta a^2 m^3/24
\]

Further, average consumers' surplus per mile is simply

\[
(38) \quad s_a(p, m) = S(p, m)/m = \frac{a - \beta p}{2\beta} - (a - \beta p)a^2/4 + \beta a^2 m^2/24
\]

Observe that \( m \) is restricted, as in (7), and this insures that the sum of the first two terms in both (37) and (38) are positive. We note in passing the not unsurprising fact that

\[
(39) \quad \delta s_a(p, m)/\partial p = -(a - \beta p) + \beta a m/4 = - q_a(p, m)
\]

where the last equality is from (9).

The net social gain resulting from the store's activities is not simply consumers' surplus; it is the total benefit received by consumers less the costs to society of providing the commodity. The social gain, then, is the sum of consumers' surplus plus consumer expenditures less the sum of transportation cost plus production cost. However, consumer expenditures are the sum of transportation costs plus the sales revenue of the store. Therefore, social gain is the sum of consumers' surplus plus the excess of sales revenue over production cost. Since the excess of the store's sales revenue over production cost is producers' surplus (profit) we observe that the social gain is merely the sum of consumers' and producers' surplus.

\[
(40) \quad g_a(p, m) = s_a(p, m) + (p - \delta) q_a(p, m) - \gamma/m
\]

\[17. \text{ Note that constant marginal utility of income was displayed for both the product differentiation and the location formulations of the utility-maximization problem when the derivation of equation (3) was discussed in Section I.}

\[18. \text{ The net social gain is equivalent to the "net social payoff" concept employed by Samuelson [11], Silberberg [13], etc. in studies of spatially separated markets.}\]
What pricing policy will maximize the gain to society from producing the commodity, given the magnitude of \( m \)? From (38) and (39) we obtain

\[
\frac{\partial g_a}{\partial p} = (p - \delta)[\partial q_a(p, m)] / \partial p
\]

Maximization of social gain thus requires that \( p = \delta \); as expected, the State Store should price at marginal cost; line \( q_a(\delta, m) \) on Figure 3 reveals average sales for given \( m \) when price is set equal to marginal cost.

If retail outlets are located at optimal frequency, differentiation of (40) reveals that we must have

\[
q_a = 4\gamma/om^2 + \beta am/12 - (p - \delta)\beta
\]

In the special case in which demand is completely insensitive to price, so that \( \beta = 0 \) in equation (9), we have \( q_a = a \), and equation (42) yields

\[
m = (4\gamma/oa)^{16}
\]

When \( \beta > 0 \) and marginal cost pricing is employed we find on equating the two alternative expressions for \( q_a \) provided by (9) and (42) that

\[
q_a = a - \beta 6 - \beta am/4 = 4\gamma/om^2 + \beta am/12
\]

Thus the optimal point on Figure 8 must be southeast of \( a' \). To read the optimal value of \( m \) off the graph we rearrange (44) to obtain \( a - \beta 6 - \beta am/3 = 4\gamma/om^2 \). The expression on the left is indicated by the dotted line on the graph and point \( b \) reveals the socially optimal \( m \) while point \( S \) indicates the corresponding value of \( q_a \). Clearly, point \( F \) at which franchise/monopoly profits are maximized involves too little product variety as well as too high a price.\(^{19}\) Furthermore, the optimally-run franchise system will have too low an average volume of sales; after all, average sales at \( c \) are one-half of the socially optimal quantity at \( S \), and average sales at \( F \) are even less than at \( c \).

With higher fixed costs, point \( S \) will be further to the southeast; thus, high fixed costs serve to reduce the socially optimal value of \( q_a \) although we should always have \( p = \delta \). Of course, it is possible for fixed costs to be so high that none of the commodity should be produced.

\(^{19}\) It is easily verified that \( b \) must be northwest of \( F \) for all admissible values of the parameters of the system if the franchise operation can function without loss.
and it turns out that \( a \) marks this cut-off point. To see why this is so, observe that at point \( a \) we have \( q_a(\delta, m) = a - \beta \delta - \beta am/4 = \beta am/4 \); therefore, \( m = 2(a - \beta \delta)/\beta a \) and \( q_a = (a - \beta \delta)/2 \). Substituting into (44) yields

\[
(a - \beta \delta)^3/3a^3 = \gamma
\]

From (39) it is easily verified that for the above values the social gain is zero. If fixed costs exceed the magnitude specified by (45), none of the commodity should be produced.\(^{20}\) It is quite possible, however, that private production will be infeasible even though production of the commodity is socially worthwhile, and it is worth noting that the bound on \( \gamma \) provided by (15) is only 4/9 of the value specified by (45).

VII. OPTIMAL ZONING REGULATIONS

What is the optimal size retail outlet, from the social point of view, given that each store maximizes its profits subject to a zoning constraint? Since the price charged by each profit-maximizing store will be affected by the width of the market assigned to it, we must consider the total derivative of (40) with respect to \( m \)

\[
dg_a(p, m)/dm = dg_a/\partial m + (dg_a/\partial p)(\partial p/\partial m)
\]

where the second equality is obtained with the aid of (41). When \( m \) is prescribed by zoning restrictions, (23) and (24) reveal that \((p - \delta) = q_a/\beta \) and \((\partial q_a/\partial p)(\partial p/\partial m) = -\beta a/\delta \). Consequently, a necessary condition for \( m \) to be optimal is that \( dg_a/dm = dg_a/\partial m - aq_a/\delta = 0 \). Differentiating (40) we now have as a necessary condition that the socially optimal \( m \) must satisfy \( dg_a/dm = -5aq_a/8 + a\sigma^2 m/48 + \gamma m^2 = 0 \). Equating the expression for \( q_a \) implied by this equation with the alternative expression provided by behavioral equation (21) reveals

\[
q_a = \beta am/30 + 8\gamma/5am^2 = (a - \beta \delta)/2 - \beta am/8
\]

The value of \( m \) satisfying this condition constitutes the most desirable density for retail outlets, given that each store charges the profit-maximizing price defined by equation (21).

To compare graphically the socially optimal \( m \), given that individual sellers maximize profits subject to a zoning restriction on \( m \), with alternative solutions, such as the \( m_f \) prescribed by a profit maximizing franchiser, it is convenient to multiply both sides of the last equation through by \( 5/2 \) to obtain \( 5(a - \beta \delta)/4 - 198am/48 = 4\gamma/am^2 \). The right-hand side has already been employed, and the left-hand side, linear in \( m \), is plotted on

\(^{20}\) Further, no real \( m \) will satisfy (44).
Figure 9. Since the two curves intersect at z, \( m_z \) denotes the breadth of the market that should be prescribed by the zoning authorities. Point Z on the \( q_a(\delta, m)/2 \) line reveals the quantity per mile that should be marketed.

The situation is necessarily sub-optimal when individual retail outlets maximize profits subject to a restriction on the size of the market they can service, as under a franchise operation or with customer-retention behavior. However, some solutions are worse than others. We always have \( m_z < m_f < m_m \). Given that each store in the franchise operation is going to maximize individual profits, the outlets will be placed too far apart and price will be too low. Optimal zoning would prescribe that the branches of a chain store should be closer together than the profit motive dictates. Alternatively, a subsidy (negative licensing fee) may be granted to induce Colonel Sanders to locate his franchise outlets at the socially optimal density.

The existence or absence of profits does not provide an adequate index of the degree of departure from the socially optimal solution. Each store may be reaping positive profits at \( m_z \), and obviously this does not indicate that the retail outlets are too far apart. If fixed costs are too large, profits may be negative at \( m_z \), and the system will not be viable without a subsidy to each retail outlet. For example, if \( \gamma \) is so large that the curve \( m = (4\gamma/aq)^{1/6} \) passes through \( M \), point \( F \) and \( M \) will coincide, and the stores will just break even when located distance \( m_m = m_f \) apart; but \( m_z \) must be less than \( m_m \), and retail outlets located that densely will experience negative profits.

VIII. SUMMARY AND CONCLUSIONS

The analysis of product differentiation has revealed some rather surprising behavioral implications. An increase in transportation costs may contribute to a reduction in the retail price charged by participants in a franchise operation or a chain store. Further, the imposition of a licensing fee on each retail outlet will not only induce an optimizing franchiser to reduce the density of retail outlets, it may also lead each franchisee to reduce his selling price. Again, if a zoning authority grants added protection against competitive forces by enlarging the zone reserved for each retail outlet, each store will find it profitable to lower price. But in each
of these cases the price reduction is achieved at the expense of a reduction in product diversity; hence, welfare is not necessarily increased. Indeed, under certain conditions subsidization of individual retail outlets—a negative licensing fee—may be appropriately applied to achieve the socially optimal degree of product diversity.

The assumption of constant marginal cost and of uniformity in the distribution of consumer tastes did not prejudice the case in favor of firms of a particular size, given that fixed costs and transportation costs are both positive. Furthermore, new entrants were assumed to suffer no cost advantage in comparison with existing firms. Nonetheless, considerable variation in the size of retail outlets might persist, even in the long run; how much variation depends upon the structure of the market. It is possible when each retail outlet engages in customer retention behavior that some buyers will be priced completely out of the market while others will suffer from too little product variety; but it is also possible under customer retention behavior for existing firms to be of the minimum size $m_m$ compatible with zero losses. The range of indeterminacy is considerably reduced but by no means eliminated under the Bertrand Postulate. In contrast, the appropriate degree of product variation is uniquely determined under a franchise arrangement. All this means that the degree of diversity in firm sizes in an industry may be more a reflection of market structure than of the shape of the average cost curve.

Although a variety of alternative market solutions may arise as a result of product differentiation, we have found that all of them are likely to involve a degree of market failure. Nonetheless, there may be strong non-economic reasons for preferring the market solution to socialized distribution. For example, as a first approximation, our cost function (10) is not an unreasonable description of textbook publishing, where $\gamma$ may denote the cost of writing, typesetting and proofreading, while $\delta$ involves the incremental printing cost per volume. We can regard the customers for elementary economic textbooks as being distributed along a continuum in terms of the degree of mathematical complexity they desire. But even though the model suggests that in these circumstances the free play of market forces might lead to the publication of too many books at too high a price, the argument of economic efficiency does not outweigh the dangers to free speech involved in state regulation and subsidization.

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21. Of course, the cost function must have the prescribed shape only within the “relevant range.” In particular, our analysis requires no modification if increasing costs eventually occur, provided that it is at a range already uneconomical in terms of shipping or other costs of differentiation.

22. This is at variance with the concept of the “survivor principle” procedure for measuring economies of scale. For an analysis of this concept, see Shepherd [12].
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