Economics 201-1, Microeconomics — Lovell
Fall, 2000-01

Assignment #7: Introduction to the Theory of the Firm

READ:

Varian, Chs. 18-22

Lovell, Ch 5 (sections 5.2 and 5.3 on business organization & accounting are optional) [This chapter is on electronic reserve and hard copy reserve.

DUE: November 2. [If you turn in your problem set by 4:00 on Wednessday, October 31 we will make every effort to return your paper to you before the quiz on November 5th].

I. Production Functions:

A firm uses two inputs, physical capital K (machinery, buildings and other capital equipment) and L years of labor to produce annual output Q. The "Cobb-Douglas" production function is

$$Q = kL^{\lambda}K^{\lambda'} \qquad (1)$$

For example, if k = 1, $\lambda = .75$ and $\lambda' = .25$ we have

$$Q = L^{0.75}K^{0.25} \qquad (2)$$

1. Suppose our firm has production function (2) above. It has 1 machine and employs 16 workers. What is output, the marginal product of labor, the average product of labor, and the marginal product of capital?

Definitions: $\partial Q/\partial L \equiv$ Marginal product of labor; Q/L is the average product of labor.

2. That is to say, if both inputs double will output double? More precisely, the production function Q(K,L) is positively homogenous of degree one if for any $\rho > 0$, $\rho Q = Q(\rho K, \rho L)$.

2a. Is Cobb-Douglas production function (2) homogeneous of degree 1?

2b. Cobb and Douglas, using data on industrial production for the U.S. for the years 1899-1922, estimated

$$Q(L,K) = 0.0156L^{0.807}\,K^{0.232}.$$

What is the degree of homogeneity of this production function?

2c. Is the production function $Q = L^{1/2} + K^{1/2}$ homogeneous of degree 1?

3. Suppose workers earn $1.00 per year and the firm has only one machine, which costs it $20 per year to operate. The firm’s production function is given by equation (2). How much will it cost our enterprise to produce 54 units of output? Now derive the firm's short-run total cost function, C(Q), showing how its total costs of operation depends on the level of output, given that it has only one machine. [Hint: First find the function showing the amount of labor required to produce Q units of output, given that our firm has only one machine.]

4. In the long run the firm may adjust the number of machines. What is the least-cost technique for producing 54 units of output, given that workers earn $1.00 per hour and machines cost $20 per year to operate? Your problem is to minimize total costs C = $1.00 L + $20 K subject to the constraint that $Q = K^{0.25}L^{0.75} = 54$.

5. Determine the firm's long-run total cost function, given that it can have as many machines as it wants at a cost of $20 per machine.

II. Cost Functions:

A firm has total cost function

$$C = 20 + 10Q - 3.75Q^2 + 0.5Q^3 \qquad (7)$$

Our firm can sell its output for p = 25 on a competitive market

6a. Determine the equations for C(Q)/Q = average total cost (ATC), [C(Q) - C(0)]/Q = average variable cost (AVC) and dC/dQ = marginal cost (MC); also find C(0) = fixed cost (FC). Now find $\pi(Q)$ = profits.

6b. Plot C(Q), R(Q) and $\pi(Q)$ on a neat graph. Plot ATC, AVC, MC and p, on another clever graph.

7. Determine the profit maximizing level of output if p = 25.

8. Prove that at that level of output for which average total cost is at its minimum, marginal cost must equal average total cost. Also, show that if labor is at the level at which the average product of labor is at its maximum then the average product of labor equals the marginal product of labor.

9. Suppose that aggregate output (Gross Domestic Product) in Econoland at time t is determined by the aggregate production function

$$Q(t) = ke^{\rho t}K(t)^{1/3}L(t)^{2/3}, \qquad (14)$$

where ρ denotes the contribution of technological progress (k is just a constant). Note that if K and L were both fixed, output would grow at rate ρ as a result of gradual improvements in techniques of production. But suppose that L grows at rate $\frac{dL/dt}{L}$ = 1.1% per annum and K grows at $\frac{dK/dt}{L}$ = 1.8% per annum and technology progresses at rate ρ = 2.2%.

9a. How rapidly will output grow? How rapidly will output per worker grow?
Hint: Take logs to the base e and determine whether the following equation holds:

$$\frac{dL/dt}{L} \quad \frac{d\ln_e Q}{dt} = \frac{dq}{dt}/q = \rho + 1/3\frac{dK/dt}{K} + 2/3\frac{dL/dt}{L} \qquad (15)$$

9b. Which is most important for increasing output per worker, technological progress or the growth of capital per worker? Explain.

10. The following table about the cost of driving a car is reproduced from the ***Statistical Abstract of the United States***, 1995. Construct the best estimate you can of the cost function for driving a car in year 1993, assuming it is the simple linear form

$$\text{Total Cost} = \alpha + \beta \text{ mileage}.$$

a. Find the marginal cost of driving the car that extra mile.
b. Carman offers to drive Wesleyan classmates to the airport (30 miles each way) in his 1987 Honda for $12. Is he making money?

Cost of owning and driving a Car, 1996
(assuming the car is driven 10,000/year[1])

Item	Unit	
Cost per mile	Cents	53.08
Variable cost	Cents/mile	10.80
Gas and Oil	Cents/mile	6.60
Maintenance	Cents/mile	2.80
Tires	Cents/mile	1.40
Fixed Cost	Dollars	4,228
Insurance	Dollars	809
License & registration	Dollars	220
Depreciation	Dollars	3,268
Finance Charges	Dollars	793

Honors Option:

Honors Option: A CES (Constant Elasticity of Substitution) production function has the form

$$q(K,L) = [\beta K^{\rho} + (1-\beta)L^{\rho}]^{\varepsilon/\rho} .$$

Here β is the distribution parameter determining the relative importance of capital and labor, ε is the elasticity of substitution, and ρ is called the scale parameter. [2]

a. What is the marginal product of labor?

b. For what values of β, ε, and ρ does this function exhibit constant returns to scale?

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[1] *Statistical Abstract of the United States*, 2000, Table 1048.

[2] This function was introduced into the literature by K. J. Arrow, H. B. Chenery, B. S. Minhas and R. M. Solow, "Capital-Labor Substitution and Economic Efficiency," *Review of Economics and Statistics (1961).*