Assignment #8: Competition & Monopoly

READ: Varian, Chapters 23-25
Lovell, Ch 5, Section 6; Ch 6, sections 6.1-6.3
[These chapters are on both electronic and hard copy reserve.

DUE: November 16.

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I. Monopoly: A firm facing the demand function

$$Q(p) = 100 - 5p$$

has total cost function

$$C(q) = 155 + 5q + q^2/20.$$

a. Determine the inverse demand function p(q), the total revenue function $R(q) = q \cdot p(q)$ and the marginal revenue function $dR(q)/dq$ for our firm.

b. Solve for the level of output and price yielding maximum profits

$$\pi = R(q) - C(q).$$

What is total revenue, total costs, total profits, profits per unit of output, marginal cost, average total cost, and average variable cost at this level of output? How large is consumer surplus?

c. Plot on a graph the demand, the marginal revenue, and the marginal cost functions. Then indicate price and quantity sold on the graph.

d. Is it true for this demand curve that at q = 0, marginal revenue equals price? Is it true that $dMR/dq = d^2R/dq^2 = 2 \cdot dp(q)/dq$ for this demand curve? Prove that the first of these two properties must hold for <u>any</u> differentiable demand curve and the second for any linear demand curve $p = a - bq$, with $a > 0$ and $b > 0$.

e. What is the elasticity of demand at the profit maximizing level of output? Recall : $\eta_p = - dq/dp \; p/q$

f. Is it true that Marginal Revenue $= p(1-1/\eta_p)$ at any level of output? Explain.

2. <u>REGULATION</u>: Congratulations, you have been appointed to the Public Utility Control Commission. The Consumer Advocate recommends that the the monopoly of question 1 should be prevented from charging a price greater than marginal cost (more precisely the monopoly must produce at the level where $p(q) = dC/dq$).

a. What price would this be? Compare the sum of consumer surplus plus profits at this level of output with that generated by the monopoly solution.

b. If your responsibility is to regulate the monopoly in order to maximize Consumer Surplus plus Profits, what price and quantity would you establish?

3. <u>TAX</u>: Congratulations, the Public Utility Commission has been abolished but the Governor has appointed you to serve as his Chief Tax Adviser. Instead of regulating the monopoly, the plan is to subject it to a 20% tax on sales revenue.

a. How will the tax influence the price and quantity sold by the monopolist? How much sales revenue will it generate?

b. Determine how the tax will affect consumer surplus and profits. Is the tax better than the public utility regulation solution in terms of maximizing the *Net Social Gain* = sum of consumer surplus plus profits = consumer surplus plus revenue - total costs? Is it better than no taxes, given that the monopoly is no longer regulated? Discuss, considering both the efficiency and the equity issues raised by the tax.

Hint: To take account of the sales tax, note that a sales tax at rate t levied on the price p_s charged by the seller means that the price paid by consumers will be $P_c = (1+t)p_s$ Substituting this expression into the demand curve yields the sales of the monopolist as a function of the price it charges (before the sales tax is added on).
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4. POLLUTION: Congratulations, the tax has been defeated in the Senate but you have been appointed Chair of the Environmental Protection Commission. Suppose that the Public Utility generates pollution costing the general public about $2.00 per unit produced. Congressman Bullhorn suggests that a tax of $2.00 per unit produced should be imposed in order to offset the social costs of pollution.
   a. How will the tax affect marginal cost and average cost?
   b. What will be the level of output produced by the monopolist if this tax is imposed.
   c. Should the government impose the tax? [Hint: You can suppose the objective is to maximize the Net Social Gain = Profits + Consumer Surplus + Tax Revenue - Pollution Costs = Consumer Surplus + Sales Revenue + Tax Revenue - Production Costs - Pollution Cost].

II. Competition: Firms in the Widget industry all have the same cost function: $C(q) = 155 + 5q + q^2/20$. The industry demand function for Widgets is $Q(p) = 1000 – 50\ p$.
   1. Determine the short-run supply function s(p) for a representative firm in this industry. How does it differ from the long-run supply function?
   2. In the long run, new firms are free to enter or leave the industry. Determine (1) the long run equilibrium price, (2) the output of the representative firm at this price, (3) the total industry sales at this price, and (4) the number of firms in the industry.
   3. Suppose a 20% sales tax is imposed on the sales of goods produced by this industry. Determine its effect on price, firm, and industry sales in the short run, given the number of firms is fixed at the level determined in b.
   4. Determine the long run effect of the sales tax, given that new firms are free to enter the industry and existing firms may leave.

III. Product Differentiation and Price Discrimination: Sellers at times market essentially the same commodity in different markets at different prices. For example, stores may sell to the "careful shopper" market at reduced prices on bargain days; spur of the moment shoppers pay the regular price. It is said that Japanese steel is "dumped" in the American market at less than cost. And the United States sells subsidized wheat abroad at a price much below what it sells for in the United States.
   a. Suppose that a firm sells its product in two differentiated markets with demand functions

$$p_1 = 10 - q_1/2$$

and

$$p_2 = 20 - q_2/4;$$

Production costs are $C(q_1,q_2) = 2(q_1 + q_2)$
   a. Find the profit maximizing level of output and prices.
   b. Under what circumstances should the government prohibit price discrimination?

IV. Management Science: Read ch 5 of Lovell, Section 5.6 before attempting this question.
   a. Use the Optimal Lot Size equation (square root rule of section 5.6.1) to check whether the consultant has indeed calculated the appropriate lot size for Table 5.4.
   b. Determine how the optimal lot sizes recorded on the table would change if the inventory carrying cost for Good A were $15, unchanged for Good B, and only $8 for Good C?
   c. Observe on that table that the setup size recommended by the consultant yields annual setup costs that are precisely equal to annual inventory carrying cost. Is this a coincidence, or can you show it must always be the case when the optimal lot size is being produced?

NOTES: While the above problem concerns the Economic Lot Size (*ELS)* for a manufacturer, the same story can be told about the that. Setting up the machines to produce a different type of gasket is expensive for it requires technical labor and involves downtime of the plant. Larger lot sizes reduce setup costs but increase inventory carrying costs.

Management Science (a journal in the Science Library) focuses on the application of economics, statistics and applied mathematics to such managerial problems as inventory control, marketing and production scheduling.