Economics 201 September 5, 2001

Problem Set #1: Supply and Demand, some Taxing Problems

Due: 8:30 AM, September 13, 2001 (Use E201-1 slot in Econ/Soc alcove)

Note: Zieqing (Simon) Shen is the TA for this course: Shen@Wesleyan.edu

ASSIGNMENT #1:

- For September 7: Varian: *Intermediate Microeconomics*, Ch 1 – Markets. Check out the Math Appendix, p 663=A1-A9 (Skip topic A.13 When we need partial derivatives we will talk about later in class)
- For September 10^{th}: Varian: Ch 16 – Equilibrium;
  Lovell, *Economics: A Calculus Introduction* Ch 3 (through p 16)
- *Plan ahead*: Prepare to ask questions about the Problem Set at the next class meeting.

Suggestions: Do the readings

Review the Notes at the end of the Problem Set before attempting Part B.

Part A: Calculus Review

1. Compute the derivatives of each of the following functions with respect to x:
   a. $f(x) = 3x$
   b. $f(x) = x^3$
   c. $f(x) = a + bx + cx^2$
   d. $f(x) = \ln(x)$ [Hint: ln(x) denotes $\log_e x$].
2. A firm's profits (π) per annum are related to its annual output (q) by the equation

$$\pi(q) = -100 + 100q - 5q^2.$$

a. Determine $d\pi/dq$.

b. Find the level of output that maximizes profits; what is the maximum level of profits (Hint: find the value of q that sets the first derivative equal to zero; check the second order conditions (sign of $d^2\pi/dq^2$).

c. If the government imposes a $200 annual license fee on this profit maximizing enterprise, what will happen to output and profits? What will happen in the long run if a license fee of $700 is imposed?

e. Forget about the license fee. What would be the effect of a 35% tax on corporate profits ?

Part B: Market Review:

1. *Competition:* The number of bushels of rye (q) that consumers demand each year depends upon price (p), measured in dollars:

$$q_d = 200 - 50p, \quad 0 \le p \le 4 \qquad (1)$$

The supply function is

$$q_s = 50p, \quad p > 0. \qquad (2)$$

a. The "equilibrium price," p^e, equates $q_d = q_s$. Determine p^e and the corresponding equilibrium quantity q^e.

Hint: First solve the problem by drawing a graph; then solve the problem algebraically for the equilibrium price and quantity by equating demand with supply.

b. Determine from the demand function an equation p(q) explaining price as a function of the quantity sold; then find a function $R(q) = p(q)q$ showing revenue as a function of quantity; then find the level of output that maximizes revenue by differentiating R(q) and setting $dR(q)/dq = 0$; verify that you have a maximum by examining d^2R/dq^2.

2. *Monopoly:* Suppose you were to succeed in monopolizing the rye market and that your total costs are $C(q) = q$.

a. find the level of output that would maximize your profits $\pi(q) = R(q) - C(q)$. What price would you charge?

b. How much profits would you make?

c. Explain why marginal revenue equals marginal cost at the profit maximizing level of output.

3. *Taxes!* Suppose that a tax of t = 50¢ per bushel is imposed by the governor on the competitive market described in question 1, assuming that competition has prevailed. Note that the governor is driving a "tax wedge" creating a gap between the price p_c paid by the consumer

and the price p_p received by the producers; i.e. $p_c - p_p = 50¢$, where p_p is the price that belongs in the supply curve and p_c is the price relevant for the determination of demand.

a. Solve simultaneously for p_c, p_p, and q^e.
b. How much tax revenue ($T = tq^e$) will the tax generate for the governor?
c. Draw a neat graph illustrating the solution that you have found algebraically; indicate on the graph p_p, p_c,, q^e, and the amount of revenue generated by the tax.

Hint: Refresh your memory by reviewing the effect of a tax in your introductory text (e.g. Samuelson and Nordhaus, 15th ed., p 65).

4. *Consumers Surplus:*

a. As a result of the 50¢ tax of question 2, how big a reduction in "consumer surplus" will be imposed on the consumers?
b. How big a loss will be imposed upon firms producing in this competitive industry?
c. How much does the combined loss of consumers and producers exceed the gain by the governor in tax revenue (this is the "deadweight loss")?
d. Show these losses in terms of relevant triangles, rectangles, etc. on a graph.

Hint: Review "Consumer Surplus" in your introductory text.

5. *Congratulations*! The governor picks you to serve as her economic consultant. She wants you to find the *revenue maximizing tax rate* t*. This is the tax rate that extracts the highest tax revenue from the market described in question 1. You must solve for the revenue maximizing tax rate, p_c and p_p, the resulting equilibrium quantity sold, and the total tax revenue collected.

You can solve this problem either graphically or with the calculus.

To solve the problem with the calculus you could first derive the equation for the equilibrium quantity sold as a function of the tax. You can do this by invoking the equation $p_p = p_c - t$ to eliminate p_c from the demand function (equation 1); then solve by equating demand with supply for the function q(t) showing how the equilibrium quantity depends on the tax rate. Second, find tax revenue as a function of the tax rate: $T(t) = tq(t)$ – this function is sometimes called the "Laffer Curve." Third, differentiate this Laffer function with respect to t and set the derivative equal to zero. (How can you verify that you have found a maximum rather than a minimum?)

Note: This is a problem in "constrained maximization;" specifically, you are to maximize the objective function $T(t) = tq^e(t)$ subject to the constraints that (1) consumers are on their demand curve, (2) producers are on their supply curve, and (3) $p_p = p_c - t$. You have solved this problem by the "method of substitution." Later, in Chapter 2, the textbook will explain how to solve such problems with "Lagrangian Multipliers." In Economics 201 the method of constrained maximization will be used in analyzing the behavior of producers and consumers.

HONORS OPTION: [Note: no partial credit for this question]

Suppose $q_d = b(a-p^c)$ and $q_s = d(p^p-c)$.

a. What is the revenue maximizing tax?
b. What is the deadweight loss from the tax?
c. What is the ratio of deadweight loss to tax revenue?
d. What determines whether the burden of the tax is shouldered primarily by the consumers or the sellers? Explain.

Consumer Surplus Note:

Consumer Surplus is the excess of the value of the commodity to consumers over what they pay for it; it is what consumers gain from trade.

British economist Alfred Marshall (1842-1924) argued that the value of a commodity to the consumer is measured by the area under the demand curve; subtracting, the amount paid for the commodity (pq), yields consumer surplus. Thus, Consumer Surplus may be measured by the integral

$$C_s = \int_0^{q^e} [p(q) - p(q^e)]dq .$$