Economics 201 September 14, 2001

Problem Set #2: Utility Analysis

Due: 11:00 AM, September 21, 2001 (Use E201-1 slot in Econ/Soc alcove)

Note: Zieqing (Simon) Shen is the TA for this course: ZShen@Wesleyan.edu; ext 5856. Review sessions are scheduled for Wednesdays, 8:00, Room 125 PAC. If you have a class scheduled for that time, let me know right away.

ASSIGNMENT #2:

- Varian, Chapters 2-6, including the appendices to chapters 4, 5 and 6 (I recommend that you read a chapter a day rather than trying to read several at one sitting. Please try to read at least one new chapter in advance of each class meeting)
- Recommended: Lovell, Chapter 4, Maximizing Satisfaction. You can download this chapter by going to the Olin Library Electronic Reserves for this course. The charts for this chapter were distributed in class on September 12, 2001
- Suggestion: Review Lovell, Chapter 3, pp 10 and 11 on partial derivatives.

Historical Note: Cambridge University Professor Alfred Marshall[1842-1924] and his followers thought the *Law of Diminishing Marginal Utility* implied the *Law of Demand* (demand curves must have a negative slope; i.e., $dq/dp < 0$); later it was determined that this condition is neither necessary nor sufficient. It was shown that a rational individual might conceivably buy more of a commodity when its price increases! And while the Law of Diminishing Marginal Utility may appeal to one's intuition, there is no obvious way to establish the proposition empirically.

Varian, p 32, question 1. Can you say whether the consumer is better or worse off as a result of the price and income changes?

1. Consider the task of allocating $10 between two individuals, Alice and Bobby. Suppose that we have been correctly informed, perhaps by divine revelation, that Alice's utility of income function is[1]

$$U_A(M_a) = \ln M_a \qquad (1)$$

while Bobby's is

$$U_B(M_b) = M_b \qquad (2)$$

a. One solution, "Fair Shares for All,"[2] is to divide the $10 equally between the two individuals. If we give $5 to each individual ($M_a = M_b = \$5$), how much utility will each enjoy? What will be the sum total of happiness, $S = U_a + U_b$?

b. Now, following Jeremy Bentham, determine how to allocate the available money so as to maximize the Sum Total of Happiness,

$$S = \ln M_a + M_b \quad , \qquad (3)$$

subject to the constraint that

$$M_a + M_b = \$10. \qquad (4)$$

Hint: You may solve this constrained maximization problem by the method of substitution; i.e. substitute from (4) into (3) or with a Lagrangian multiplier

c. How would the allocation change if we had $20 to allocate between Alice and Bobby? Does this seem reasonable?

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[1] Here ln Y denotes the log to the base e of Y

[2] This was the slogan used to generate public support for rationing in Britain during World War II.

Class Discussion Questions –

1. *"Scientific Socialism:"* Do you believe that the poor have a higher marginal utility of income than the rich. If so, should we redistribute funds from the rich to the poor in order to achieve the greatest good for the greatest number — or at least to maximize the sum total of utility?
2. *Optimal Population Policy:* If every citizen in the mthical country of Econoland has utility function $U_i(X_i) = \ln(X_i)$ and there are a fixed number of resources X, what is the optimal population size if the objective is to maximize total satisfaction, $S = \sum U(X_i)$? What if the objective is to maximize average satisfaction?

3. Consider the following utility functions:

Alice:	$U_a = 3X + Y$
Baker:	[3] $U_b = \ln X + \ln Y$
Charley:	$U_c = XY$
Debby	$U_d = X + Y^{1/2}$

a. Plot two indifference curves for each individual [Suggestion: Use a separate graph for each utility map.]
<u>Example</u>: All three of the following alternative consumption bundles are on the same indifference curve for Alice because each would provide her with 9 utiles of satisfaction:

	#1	#2	#3
X:	3	1	0
Y:	0	6	9

Therefore, these three points are all on Alice's U = 9 indifference curve.
<u>Hint</u> Two of these individuals have the same indifference curves

b. Do any of the indifference curves you have plotted violate the convexity assumption (consider Varian, page 47)?

c. For each of the four utility functions, please find, using the calculus, the marginal utility of X, the marginal utility of Y and the marginal rate of substitution, all evaluated at the point <1,2>; i.e., find $\partial U/\partial X, \partial U/\partial Y$, and $-dY/dX|_u$

d. Does the marginal utility of X, $\partial U/\partial X$, depend on the quantity of Y consumed? Find out by calculating $\partial U^2/\partial X \partial Y$ for each of the three utility functions. (Observe that Charlie's utility function, unlike Alice's and Baker's, is not separable — does this explain what you found?)

e. Which of the above utility functions satisfy the law of diminishing marginal utility?

<u>Definition</u>: The good X_i is subject to "*diminishing marginal utility*" if and only if $\partial^2 U/\partial X_i^2 < 0$.
<u>Note</u>: Classical economists stressed the importance of the Law of Diminishing Marginal Utility. In the most popular economics textbook of its day, Alfred Marshall explained: "...the total utility of a thing to anyone increases with any increase in the stock of it, but not as fast as his stock increases.. In other words, the marginal utility of a thing diminishes with every increase in the amount of it he already has."

4 Charley has utility function $U_c = XY$.

a. Suppose we give Charley $100 to spend on goods X and Y. Suppose that a unit of X costs $4.00 and the price of a unit of Y is $2.00. How much of X and of Y will Charley purchase, assuming he acts to maximize his total utility?
Hint: Charley must solve the following constrained maximization problem:
Maximize U=XY

[3] Here lnX denotes the log to the base e of X, or $\ln_e X$; i.e. $e^{\ln X} = X$.

subject to the budget constraint that

$$\$4.00X + \$2.00Y = \$100.$$

To solve the problem by the method of substitution, first solve the income constraint for Y as a function of X. and substitute into the utility function But why not try the Lagrangian technique? Check out Varian, page 93.

b. More generally, suppose that Charley has income M, that the price of X is p_x and the price of Y is p_y. Solve, under the assumption that Charley is a utility maximizer, for the demand function for X showing how the consumption of that good depends upon its own price, on income, and on the price of the other good.

c. Solve for Charlie's "*indirect utility function*" showing his utility as a function of income and prices under the assumption that he always maximizes utility. This is the function $U^I = V(p_x, p_y, I)$ described in Lovell, Chapter 4, page 12.

d. Find Charlie's "*expenditure function*" showing the level of income he would have to have in order to achieve utility level U. That is, the function $M = E(U, p_x, p_y)$ described in Lovell, Ch 4, page 13.

e. Application: Suppose initially $p_x = \$1.00$, $p_y = \$2.00$ and M = $10. Find X, Y, and U [Hint: use Charlie's demand functions and his indirect utility function.] Now suppose that an excise tax of $1.00 increases p_x to $2.00 while p_y is unchanged. How much X and how much Y will Charlie consume and what will be his new level of utility?

f. Now use the expenditure function to find out how much Charlie's income would have to increase in order to leave him with the same level of utility as before the price increase. If his boss were to give him precisely this increase in income, how much X and how much Y will Charlie consume? Note: this *compensating income increase* is a possible measure of the loss of consumer surplus Charlie suffers as a result of the tax.

g. As an alternative to the $1 excise tax on X, suppose Charlie is required to pay an income tax that is so large that it lowers his utility to the level you found in e. Would the income tax yield more or less revenue than the sales tax? [Hint: use the *expenditure function* with $p_x = \$1.00$, $p_y = \$2.00$ and the resulting level of U you found in f] Note: this "equivalent income reduction" is another possible measure of the loss of consumer surplus that Charlie suffers as a result of the sales tax. Would the government collect more revenue with the excise tax on X or with the income tax which causes the same loss of utility as the excise tax?

3. Suppose that Debbie has the following utility function: $U^D(X,Y) = X^{1/2}Y^{1/2}$.

a. Does her utility function satisfy the Law of Diminishing Marginal Utility?

b. Does her demand functions for X and for Y differ from those of Charlie, who had utility function: $U^C(X,Y) = XY$.

c. Can you think of an experiment that could, even under ideal conditions, be used to determine whether an indivdual with the same demand function as Charlie has diminishing, constant, or increasing marginal utility of X?

d. A monotone increasing transformation is a function T(x) such that dT/dx > 0. Can you find a monotone transformation T(U) such that $U^D(X,Y) = T[U^C(X,Y)]$.

Proposition: If there exists a monotone transformation such that $U^D(X,Y) = T[U^C(X,Y)]$, then the two utility functions will generate the same demand functions.

HONORS OPTION:

1. Prove the above proposition
2. Derive the demand function for X_1 implied by the Stone-Geary utility function (No partial credit):

$$U(X_1, X_2) = (X_1 - \beta_1)^{\alpha_1}(X_2 - \beta_2)^{\alpha_2}$$

$$for \;\; X_1 > \beta_1 \;\; and \;\; X_2 > \beta_2; \quad 0 \;\; otherwise.$$