Economics 271- Lovell September 21, 2001r

Problem Set #3: DEMAND FUNCTIONS, ETC.

READ: Varian, Chs. 7, 8, and 9

Due: Friday, September 28, 11:00 AM. But if you put your assignment in the White Lock Box in the Econ/Soc Alcove by 4:30 PM on Wednesday, September 26th, we will do our best to return it to you before the quiz on Monday, October 1st.

1. Definition: A function $Y = f(X_1,X_2)$ is homogeneous of degree k if and only if for any coefficient ρ,

   $\rho^k f(X_1,X_2) = f(\rho X_1,\rho X_2,)$.

   Example: if $k = 0$, $\rho^k = 1$ and $f(X_1,X_2) = f(\rho X_1,\rho X_2,)$.

   Determine which of the following demand functions is homogeneous of degree 0 in income and prices:

   a. $X(p_x,p_y,M) = 5 + 3p_y - 4p_x + 3M$
   b. $Y(p_x,p_y,M) = (p_x + M)/p_y$
   c. $\ln Y(p_x,p_y,M) = 10 + 3\ln M - \ln p_y$

   Query: If a demand function is not homogeneous of degree zero in money income and prices, can it describe the behavior of a utility maximizing individual? Explain.

2. Consider the following utility functions:

   Albert: $U^A = 2\ln X + \ln Y$;
   Baker: $U^B = \ln X + Y$; and
   Cynthia: $U^C = \ln X + 100 - 1/(Y+1)$ (Note ln X is the log of X to the base e.)

   a. Which utility function is "homothetic" (Varian, page 101). Explain. [Suggestion: For each utility function find the equations for the marginal utility of X and the marginal utility of Y; then calculate MU_x/MU_Y to find the equation for the marginal rate of substitution (MRS) as a function of X and Y. (The MRS tells us how much Y we would have to give up to obtain an additional unit of X.) Inspect the equation to see whether the MRS depends only on X/Y?]
   b. Explain the special feature of the Engel's curve for any utility function that is homothetic [The Engel curve shows how consumption changes with income, given prices,?

3. Verify that the Slutzky equation is satisfied for the values calculated in Problem Set 2 for the utility function $U_c = X_1X_2$

4. George has to choose between putting time in on the job to earn money and leisure. His utility function is $U = M + 40L^{1/2}$, where M is the things he buys with money and L is hours of leisure per day. Since there are 24 hours in the day, hours worked $H = 24 - L$. The wage is \$5.00 per hour, so his income is $M = \$5(24-L)$.

   a. How many hours will George work each day? Solve this constrained maximization problem by the method of substitution. Then solve again using a Lagrangian multiplier.
   b. If an income tax of 20% is imposed, so his after tax hourly wage is reduced to \$4, how many hours will he work each day?
   c. If his late uncle leaves a substantial inheritance for him in a trust fund that yields a daily income of \$25, how will George's work effort be affected?

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Class Discussion ~ Not to hand in.

Consumer Surplus Ambiguity: Here is a contrived example suggesting that the concept of consumer surplus may be ambiguous.

George has an income of \$10.00, bananas cost 1.00 and apples cost \$2.00. He chooses to consume 3 apples and 4 bananas. An interviewer hopes to determine by asking appropriate questions how much consumer surplus he enjoys as a result of the opportunity to purchase bananas.

a. Draw his budget line on a graph with the quantity of bananas plotted on the abscissa and apples on the ordinate. Mark with an a the point <4,3> at which he choose to consume.
b. George tells our interviewer that if he was deprived of the opportunity of eating bananas he would suffer a loss of utility unless his income were increased to \$13.00, in which case he would consume 6 1/2 apples, zero bananas, and be exactly as well off as at point a. The interviewer concludes that George enjoys a consumer surplus of \$3.00! Mark with a b on your graph the point <0,6.5>; draw the indifference curve that passes through both points a and b.
c. George tells a second interviewer that he would be indifferent between consuming 5 apples and zero bananas and as an alternative giving up \$2.00 in order to retain the privilege of buying bananas at a price of \$1.00. Draw in the budget line that would confront George if his income were reduced to \$8.00. Explain why the indifference curve going through point <0,5> is just tangent to this budget line. The second interviewer concludes that George is enjoying a consumer surplus of \$2.00.

d. Do the answers to the two interviewers reveal that George is inconsistent (i. e., do his answers imply that his indifference curves must cross on have a positive slope)? Does either interviewer have the right estimate of the amount of consumer surplus George enjoys? Or is there an inherent ambiguity in the concept of consumer surplus? Explain.

Notes: If the marginal utility of the other good is constant (i.e., the indifference map is quasi linear), there is no income effect and the two definitions of consumer surplus yield the same estimate.)

An interesting retail innovation is the "Price Club" — You have to pay an annual membership fee for the privilege of shopping at the "club."

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The "Compensated Demand Curve:" When we derived the demand function from the utility function we assumed that money income was constant. Suppose instead that when prices change your income is "indexed"; that is to say, your employment contract specifies that your money income will be adjusted so as to allow you to keep the same level of utility as before the price change – then we would have your constant real income or "compensated" demand curve.

Substituting the expenditure function $M(U,p_x,p_y)$ (equation 37. p 13 of Ch 4) into the (constant money income) demand function $X_2(p_x,p_y,M)$ yields $X_2(p_x,p_y,M(U,p_x,p_y)) = X_2^c(U,p_x,p_y)$ the "constant real income" or "compensated" demand function for X_2. It explains how the consumer will respond when prices change if the employer compensates the worker for the effects of inflation by raising money income just enough to hold utility constant. Because real income is being held constant, the compensated demand functione captures only the substitution effect; it eliminates the income effect; therefore, the constant real income demand curve will always have a negative slope:

$$\partial X_2/\partial p_2|_U < 0.$$

The Slutzky equation: The Slutzky equation states that the effect of a small change in price upon the quantity of a good demanded, given money income, is equal to the sum of the "substitution effect" plus the "income effect."

$$\partial X_2/\partial p_2 = \text{substitution effect} + \text{income effect}$$
$$= \partial X_2/\partial p|_U \quad + \quad - X_2 \partial X_2/\partial M$$

On the left of the Slutzky equation we have $\partial X_2/\partial p_2$, which is the reciprocal of the slope of the constant money income demand curve (see equation 8 above). On the right of the Slutzky equation are two terms adding up to $\partial X_2/\partial p_2$:

#1: The first of these two terms, the "*substitution effect*," is the reciprocal of the slope of the compensated demand curve. It is always negative.

#2 The second term on the right is the "*income effect*," as measured by $- X_2 \partial X_2/\partial M$. Here $\partial X_2/\partial M$ is the slope of the Engel curve and X_2 is the quantity consumed of the commodity.

It may help to understand the income effect intuitively to note that an increase of $1.00 in the price of X_2 is roughly similar to a reduction in income of X_2 dollars *if* you were to continue spending p_2X_2 on that good. And a decrease in income of X_2 dollars would imply a reduction in consumption of $-X_2 dX_2/dM$, which is the income effect of the price increase.

To sum up, the Slutzky equation explains that the slope of the constant money income demand curve includes both the *substitution effect* of price changes on consumption given real income plus the *income effect*.

These concepts explain what determines the slope of demand curves. For a *normal good*, $\partial X_2/\partial M > 0$ and so the income effect is negative. Thus, the reduction in consumption brought about by the substitution effect of an increase in the price of a *normal* good is reinforced by the income effect, contributing to a still further reduction in consumption. For an *inferior* good, $\partial X_2/\partial M < 0$, which makes the income effect positive; this means that the slope of the compensated demand curve is less than the slope of the constant money income demand curve. And in the extreme case of a *Giffen Good*, the substitution effect is more than offset by the income effect, and a price increase causes an increase in consumption, given money income!

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<u>Study Questions</u> (not to hand in): Consider the following propositions. Prove those statements that are true; demonstrate that the incorrect statements are false. (You may show that a statement is false by providing an appropriate counter example. Use either a graph or the calculus.)

#1. Show that the demand functions for each of n commodities consumed by a utility maximizing individual must be homogeneous of degree zero in all prices and income.

#2. A Laspayres price index overstates inflation while a Paasche price index understates inflation. Why? Because consumers successfully substitute away from the commodities that increase most in price.

#3. Suppose that empirical research establishes that a consumer who purchases two commodities, X_1 and X_2, has demand function $X_1 = M/2p_1$ and $X_2 = M/2p_2$. One researcher concludes that since these demand functions might have been generated by the utility function $U = (X_1,X_2)^{0.5}$ the empirical evidence establishes that our consumer must have diminishing marginal utility. A second researcher observes that since these demand functions could have been generated by the utility function $U = (X_1X_2)^2$ our consumer must violate the law of diminishing marginal utility. Who is right? Can you construct an experiment that might serve to refute the law of diminishing marginal utility?

Note: Nobel Laureate Paul Samuelson argued that the concept of diminishing marginal utility is "non-operational" in that it cannot be rejected empirically even under idealized experimental conditions.

#4 If two individuals have utility functions such that $U_1(X_1,...,X_n) = g[U_2(X_1,...,X_n)]$, for all X_i, where $g(U_2)$ is a monotonically increasing transformation, then they must have the identical demand functions. Note, the function g amounts to a renumbering of the indifference curve that does not affect their ranking.

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Lagrangian Example (from class, 9/19): There are three steps to maximizing the utility function $U = kX + \ln Y$ subject to the income constraint $p_xX + p_yY - M = 0$

Step 1: We form the Lagrangian expression

$L(X,Y,\lambda) = kX + \ln Y - \lambda(p_xX + p_yY - M)$,

a function in three unknowns but no constraints. The new parameter λ is know as the Lagragian multiplier.

Step 2: Find the necessary conditions for a maximum of $L(X,Y,\lambda)$ by setting its derivatives with respect to each of its three arguments equal to zero

#1 $\partial L(X,Y,\lambda)/\partial X = k - \lambda p_x = 0$

#2 $\partial L(X,Y,\lambda)/\partial Y = 1/Y - \lambda p_y = 0$

#3 $\partial L(X,Y,\lambda)/\partial \lambda = p_x X + p_y Y - M = 0$

Note that the third condition just reproduces the income constraint.

Step 3: Solve the equations for the max:

From #1, $\lambda = k/p_x$

Substituting into #2, $Y = 1/(\lambda p_y) = p_x/(k\ p_y)$.

From #3 we now find that $X = M/p_x - 1/k$.

But note that this solution is invalid if it implies that more than income should be spent on Y. Therefore, if $p_x/k > M$, $Y = M/p_y$ and $X = 0$, so we could write

$X = \max(0, M/p_x - 1/k)$ and $Y = \min(M/p_y, p_x/(k\ p_y))$.

Comment: Economists usually use the Lagrangian technique rather than the method of substitution for several reasons. One advantage is that the procedure treats the unknowns symmetrically. Also, it generalizes easily to the case of more than one constraint by just adding additional Lagrangian multipliers.