Problem Set #6: <u>DECISION MAKING UNDER UNCERTAINTY</u>

READ: Varian, Chapter 12. Chapter 13 is a valuable option which is highly recommended for students who have studied probability and statistics (e.g., Econ 270~Econ200) and who are interested in their application to investment in the stock market.

DUE: 11:00 noon, October 26,.

Recommended: Read about decision making under uncertainty in one of the alternative textbooks on reserve: For example: Baumol (Ch. 17, Neumann-Morgenstern). Henderson-Quandt (pp 42-47, Choices Involving Risk) Future lawyers may want to check out a classic paper by Richard Freeman, "Legal Cobwebs, the Changing Market for Lawyers," *Review of Economics and Statistics*, May, 1975.

1.   *Expected Value* Computations: Insight into the problem of decision making uncertainty was provided by the Swiss mathematician Daniel Bernoulli[1700-1782], who argued that the utility of an uncertain prospect (e.g., a lottery ticket) equals the expected value of the utility of the outcomes. The basic premise is that rational decision making under uncertainty involves the maximization of expected utility. Bernoulli also assumed that utility was equal to lnY.

Example: At the ball park a professional gambler offers to pay you $4.00 if the Red Sox win; if they don't win, you are out $5. If you assign subjective probabilities of 1/2 to each of the two possible outcomes of this experiment, the expected value of the game can be calculated as follows.

Outcome	Subjective Probability	× Random Variables	= Product
Win	1/2	+ $4.00	$2.00
Lose	1/2	- 5.00	-2.50

   Expected value of the game: -0.50

Concepts: A "*random variable*" is a number assigned to a conceivable output of an experiment – e.g. $5. A "*fair game*" has zero expected value. The "*subjective probability*" of an event does not have to be objectively determined on the basis of repeated experiments.

1a.   What is the expected value of this game if your subjective probability of a Red Sox victory in 0.75.

Expected Utility Calculations: Three friends at the ball park consider the gambler's offer. All believe the Red Sox have a 50/50 probability of winning. The following table provides relevant information about the utility functions of these three friends and a stranger:

Income Y	Mr. Risk Neutral	Ms Risk Lover	Mr. Risk Adverse	Mr. Stranger
Y	10 utiles	10 utiles	10 utiles	22 utiles
Y-$5	5	9	4	10
Y+$4	14	13	12	26
Expected Utility of the Game	9.5	_____	_____	_____

Note that each baseball fan will have an income of Y yielding utility of 10 if they turn down the gamble.
The expected utility for Mr. Risk Neutral is equal to $1/2 \times 5 + 1/2 \times 14 = 9.5$. Since this is less than the utility of 10 he will enjoy with certainty if he abstains from gambling, he will not take the bet.

1b.   Complete the bottom row of the table. Then determine which of these individuals will want to take the bet rather than enjoy the 10 utiles of satisfaction with certainty by not betting. Explain why.

2.   The great English economist Alfred Marshall[1842-1924] argued in his *Principles of Political Economy* that the law of diminishing marginal utility implies that "gambling involves an economic loss, even when conducted on perfectly fair and legal terms." To demonstrate his argument he presented the following illustrative example: "For instance, A man who having 600 pounds makes an even bet of 100 pounds, has now an expectation of happiness equal to half that derived from 700 pounds and half that derived from 500; and this is less than that derived from 600 pounds because by hypothesis the difference between the happiness got by 600 and 500 pounds is greater than the difference between the happiness got from 700 and 600."

Comment: Marshall relied on the assumption of diminishing marginal utility in reaching his conclusion that gambling involved an economic loss. Most modern economists, starting with the assumption that people are rational maximizes of expected utility, would argue that those who gamble (like Mr. Risk Lover) must have increasing marginal utility!

2a.   Suppose that Harold, in contradiction to Marshall's assumption of diminishing marginal utility, obtains 1 utile of satisfaction from 500 pounds, 2 utiles from 600 pounds, and 4 utiles from 700. Would Harold prefer 600 pounds with certainty to the fair 100 pound gamble described by Marshall? Explain carefully. [Hint: Evaluate the expected utility of the gamble]

2b.   Marshall went on to argue that "theoretically fair insurance is always an economic gain." Now suppose Harold has 700 pounds but he believes there is a 50% probability that he might lose 200 pounds from a fire; would it be wise for him to pay 100 pounds to insure himself against the loss? Explain.

Class Discussion: Suppose that Alfred Marshall and co-authors John von Neumann & Oscar Morgenstern (looking down from the heavens) observe someone gambling at a race track. Would they conclude (1) she enjoys horses, (2) she has increasing marginal utility of income, or (3) she is not rational? Explain!

3.   Price Stabilization: Suppose that the demand for wheat in Never-Never Land is $q = 200 - 50p$. Even numbered years are good, but the other half of the time crops are "bad." In the drought years farmers produce only 50 units but in good crop years they produce 150.

3a.   The price in a good crop year will be $\$\_\_\_\_$ and total farm revenue will be $\$_____$; in a bad year the price will be $\$\_\_\_\_$ and total farm revenue will be $\$\_\_\_\_\_$. In a bad year consumer surplus is $\$_____$ while in a good year it is $\$_____$.

Moral: Good weather is good for the consumer but bad for farmers.

3b.   Suppose the government, concerned about the plight of the farmers, moves to stabilize the grain market by establishing an "Ever-Normal-Granary," offering to buy wheat from anyone at $1.99 or sell it at $2.01 (round off to $2.00 for ease of calculation). In a good crop year the government will buy _____ units of wheat, the price will be $\$\_\_\_\_\_$, consumer surplus will be _____, and total farm revenue will be _____. In bad years the government will sell _____ units, consumer surplus will be _____ and total farm revenue will be $\$_____$. Does stabilizing commodity prices stabilize farm income? Assuming risk neutrality, who benefits and who loses from this policy? Suppose that a special interest group argues that the government should stabilize the price at $2.20 rather than $2.00 per bushel. Who would gain and who would lose from the higher support price?

<u>Moral</u>: Price stabilization may not stabilize the farmer's income.

c.   If the demand curve had been of the form q = 150/p, the price elasticity of demand would be unity. Would the farmers benefit from the policy of stabilizing the price of wheat at $2.00? Would the Ever-Normal Granary overflow?

<u>Moral</u>: *Price stabilization can lead to a growing agricultural surplus even if it does not influence average farm revenue or average price.*

4.   Private Speculation: Suppose, as in the preceding problem, that the demand for wheat is q = 200 - 50p, that in good crop years farmers produce 150 units and in bad crop years they produce only 50. Suppose also that the government decides not to intervene in the wheat market.

4a.   A speculator observes the fluctuations in the price of wheat. She decides to buy 7 units of wheat in good years and to sell 7 units in years of drought. As a result of this speculative activity, the price of wheat will be $_____ in years of good harvest and only $_____ in bad years. Her profits average out to $_____ per year (neglecting storage, interest, inventory shrinkage and spoilage).

<u>Moral</u>: *Private Speculation may help to stabilize price fluctuations.*

Honors Option: A Wesleyan graduate suspects the speculator could increase profits by applying the calculus to determine the optimal quantity of wheat to buy in good harvest years rather than relying on the lucky number seven. Find the optimal extent that the speculator should enter the market, taking into account the effect of her activities on the price of wheat in both good and bad years.

4b.   Innovators are often imitated, and speculation is no exception to this rule. If others continue to engage in speculation as long as profits are to be made, what will happen to the wheat market?

<u>Moral</u> — *Private speculators may serve the socially useful function of stabilizing markets, provided the government resists the temptation to intervene.   Speculators only temporarily earn abnormally high profits.*

For Class Discussion

1.   Joe Oilman is down to his last million dollars.   He contemplates spending the million on drilling an oil well. There is one chance in five that he will hit a gusher, and if he strikes oil he will get $10 million in profits. However, if the hole is dry he will be out the million invested in the project.
   a.   What is the expected value of the gamble?
   b.   If his utility function is $U = W^{0.5}$ what will be the expected utility from the gamble.
   c.   If Joe is an expected utility maximizer, will he drill for oil? Explain!
   d.   Joe's brother Moe is confronted with exactly the same choices. His utility function is $U_m = W$; will Joe drill for oil?

2.   Congratulations, you have inherited $5,000 from your late uncle's estate. Your trusted investment counselor advises that you can make an investment in a project that has a 50% chance of paying $10,000. And there is a 50% probability that you will get zero.
   a.   What is the expected value of the investment?

      b.   Suppose that you scale your utility function so that U(0) = 0 and U($10,000) = 10 utiles. What is the expected utility from the investment.

      c.   If you decide to make the investment, does this imply that you have increasing or diminishing marginal utility of wealth?

3.   Forget about the earlier project — your investment counselor reports that you spent so much time evaluating the prospect that you missed the opportunity. However, there is another project costing $3,000. It offers a 45% probability of winning $10,000 and a 55% probability of zero. Is this an "actuarially fair" proposition? How will you decide what to do?
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The Market for Lemons, an Application of the Theory of *Adverse Selection*. This optional question is recommended for students who have had Econ 270 (Econ 201). The following analysis is based on a famous paper by George Akerlove, a winner of this year's Nobel Prize in Economics!

You are planning to buy a second hand vintage automobile — a 1985 Edsel. Cars of this make and model year have a 25% probability of being lemons. If a car is a lemon, the probability that its owner (recognizing the problem) will sell it this year is 3/4; if the car is not a lemon, the probability that its owner (recognizing its fine qualities) will sell it is only 1/4.

(Note: Information is asymmetric — sellers know the quality of the car they have been driving but potential buyers cannot determine whether a car is a lemon or not until after they buy it).

a.   What is the probability that a 1985 Edsel selected at random (e.g., seen in the Super Market parking lot) is both on the market and a lemon.

b.   What proportion of all 1985 Edsel cars will be sold this year?

c.   What is the probability that the 1985 Edsel for sale on Lovell's Used Car lot is a lemon?

d.   If a lemon Edsel is worth $100 and a non-lemon Edsel is worth $1,100, what is the expected value of the average 1985 Edsel car on the highway today? What is the expected value of a 1985 Edsel car that is for sale on the second-hand market?

e.   A randomly selected Edsel is totaled! That's bad, but the car was fully insured. That's good, but the insurance company pays the retail value of the car, which is estimated by the bluebook to be $600. What is the expected monetary loss experienced by the owner of a good Edsel as a result of the damage to the car, assuming no other monetary losses or injuries? Explain carefully.

Reference: George A. Akerlove, "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, August 1970, pp. 488-500.