

### Problem Set #9: Imperfect Markets

Due: November 30, 11:00 AM

**READ:** Varian, Chapters 26, 27 and 28. Lovell Ch 6; Lovell Ch 7.3 on Wage Determination, etc.

Note: the graphs for Ch 6 and Ch 7.3 were distributed in class on November 14<sup>th</sup>.

1. **Monopolistic Competition:** As with the example in Chapter 6.3, the  $i$ th Company sells its product in a monopolistic competitive market characterized by product differentiation. The demand function is (see equation 30 of 6.3)

$$q_i = 10(11 - p_i + 0.75 \bar{p})/n^{0.5},$$

where  $p_i$  is the price it charges,  $\bar{p}$  is the average price charged by all the firms in the industry,  $q_i$  is annual sales, and  $n$  is the number of firms in the industry. There is free entry and exit from the industry. Production costs for each firm in the industry are given by the function  $C = 64 + 4q_i$ ; i.e., there are fixed costs of \$64 and marginal cost = average variable costs = \$4. There are 100 firms in the industry.

- a. If all the other competing firms are charging  $\bar{p} = \$12$ , what is the best price for the  $i$ th company to charge in order to maximize its profits? Is the industry in long run monopolistic-competitive equilibrium?

The newly elected governor has imposed a licensing fee of \$64 on each firm. Subsequently the number of firms declines to 25 but the price remains at \$12.

- b. Would the  $i$ th company be better off if every firm in the industry charged \$18.00? Explain. Hint calculate the resulting quantities and profits if every firm charges \$18.00. If all the other firms are charging \$18.00, what is the best price for the  $i$ th company to charge?
- c. Would this industry be in long run monopolistic-competitive equilibrium with  $p_i = \bar{p} = \$18$  and  $n = 25$ ?
- d. Is this industry be in long run monopolistic-competitive equilibrium with  $p_i = \bar{p} = \$12$  and  $n = 25$ ? Explain.
- e. The governor obtains  $25 \times \$64 = \$1,600$  revenue from the licensing fee. Is anyone hurt by the imposition of the licensing fee? Explain?

2. **DUOPOLY ~ No Product Differentiation:** Two firms produce an undifferentiated product for which the demand curve is

$$q = 200 - p \tag{1}$$

where  $q = q_1 + q_2$  is the sum of the output of the two firms in the industry. Suppose that the cost function of Firm 1 is simply

$$C_1(q_1) = 20q_1, \tag{2}$$

where  $C_1$  denotes the total cost incurred by Firm 1.

Total profits of the firm are then

$$\pi_1(q_1, 0) = [200 - (q_1 + q_2)]q_1 - 20q_1. \tag{3}$$

- a. For what combinations of  $q_1$  and  $q_2$  will  $\pi_1(q_1, q_2) = 0$ ? Plot this “isoprofit = 0” curve on a graph with  $q_2$  on the horizontal axis and  $q_1$  plotted vertically.
- b. Find the value of  $q_1$  that maximizes  $\pi_1(q_1, 0)$ . What is the resulting profit?
- c. Find the value of  $q_1$  that maximizes  $\pi_1(q_1, 100)$  and the resulting profit. Indicate on the graph the general shape of an isoprofit curve showing all combinations of  $q_1$  and  $q_2$  yielding this level of profit for Firm 1.
- d. If as suggested by A. A. Cournot (1838), Firm 1 believes that its behavior will not affect Firm 2’s output, then the profit maximizing output of Firm 1 is a function of the given output of Firm 2. Determine this “reaction curve” and plot it on the graph.
- e. Firm 2’s total costs are  $C_2(q_2) = q_2^2$ . Determine the equation for profits  $\pi_2(q_1, q_2)$ .

- If Firm 2 regards Firm 1's quantity as fixed, what will be its reaction function?
- Determine, with the reaction curves of Firms 1 and 2 as given by questions d and e the equilibrium quantities that will be sold by the two firms. Is this "Cournot equilibrium" stable?
  - If Firm 1 knew Firm 2's reaction curve, what should it produce in order to maximize profits?
  - Suppose that Firm 2 always follows the alternative strategy of adjusting its price so as to always maintain a particular *market share*, say  $1/3$ ; i.e.,  $q_2 = 1/3 q_1$ . Then if Firm 1 knows this behavior pattern, how should it respond (indicate on the graph) in order to maximize profits?

3. **DUOPOLY ~ Product Differentiation:** Two firms produce differentiated products whose demands are interrelated. Specifically,

$$q_1 = 100 - p_1 + 0.5 p_2 \quad (4a)$$

$$q_2 = 100 + 0.5p_1 - p_2 \quad (4b)$$

Thus the revenue of Firm 1 is

$$R_1(p_1, q_1) = 100 p_1 - p_1^2 + .5p_1 p_2. \quad (5)$$

Let us suppose that  $C_1(q_1) = 20 q_1$

- Determine the profits of Firm 1 as a function of prices; i.e., find the equation for  $\pi_1(p_1, p_2) = R_1(p_1, p_2) - C_1$
- Determine the reaction curve giving the price of Firm 1 as a function of the price of Firm 2, if Firm 1 acts on the assumption that Firm 2 will not change its price.
- If Firm 2 has cost function  $C_2(q_2) = 20q_2$ , then its reaction curve will be of identical form to that of Firm 1. Plot the two reaction curves and determine the equilibrium prices, quantities and profits.
- Joint Profit Maximization: Suppose Firm 1 and Firm 2 decide to collude in order to maximize total profits. What prices would they charge and what quantities would they sell. Determine their joint profits.
- Price Leadership: Suppose that Firm 1 is the acknowledged "*price leader*;" That is to say, Firm 2 always follows by charging the same price as Firm 1, whatever that may be. What is the optimal price for Firm 1 to charge?

6. **GAME THEORY:** Congratulations, you have been promoted to President of the Ajax manufacturing company. Ajax sells in a duopolistic market in competition with the Bjax company. Duopolists Ajax and Bjax are both considering an expansion of their capacity. The following payoff matrix shows how their profits (in millions of dollars) depend on their decisions (The first number of each pair is the profit of Ajax; the second *number* is the profit of Bjax):

	B has low capacity	B has large capacity
A has low capacity	\$8, \$5	\$5, \$4
A has large capacity	\$6, \$4	\$14, \$3

- Define what is meant by a "dominant strategy". Does the Ajax company have a dominant strategy? Explain.
- Define what is meant by a "Nash equilibrium." Is there a Nash equilibrium for this game?
- Will you expand your plant capacity, Mr. President? Explain your decision.

**Honors Option Challenge:**

- Returning to question #4, suppose that Firm one achieves monopoly control over the industry by buying out Firm 2. How much will the monopoly produce at each factory in order to maximize profits? What will be the level of profit and the level of consumer surplus when the industry is monopolized?
- Is it possible for price leadership to lead to prices that would be higher than would prevail under a collusive joint-profit-maximizing regime? Explain.