Economics 201 — Mike Lovell
November 5, 2001

Post Mortem ~ Quiz #2

Part I:

1. If $q = 500{,}000p^{-1/2}M^{2/3}$, then the price elasticity of demand is
   $\eta = 1/2\ 500000\ p^{-1.5}M^{2/3} \times p/q = 1/2\ [500000\ p^{-0.5}M^{2/3}/p] \times p/q$.
   But the expression in brackets is q! So everything cancels out leaving $\eta = 0.5$
   Similarly, $\eta_m = 2/3$.
2. The *arc* elasticity concept is relevant: η = 7%/10% or 8%/10% = 0.7 or 0.8
3. Since $\$10{,}000 = \$12{,}100/(1+10\%)^2$, the internal rate of return is 10%; i.e., that rate equates the future payment(s) with today's investment.

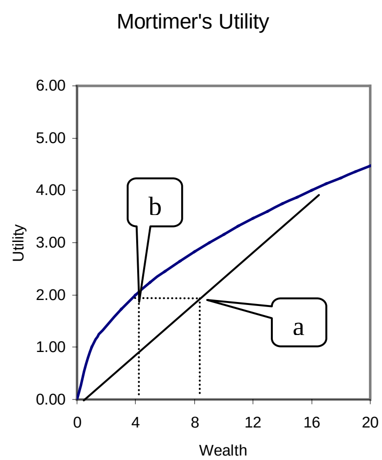
Part II:

1.

a. E(W) = ½ 0 + ½ \$16 = \$8

b. E(U) = ½U(0) + ½ U(\$16) = ½ 0 + ½ 4 = 2.

c. He is risk adverse because the U[E(W)] = U(\$8) = 2.828 > E(U); i,e, this holds because his utility function obeys the law of diminishing marginal utility.



d. Mortimer would be willing to pay \$12 in order to be insured against the risk of being robbed because the utility of the remaining \$4 that he would have (at point b) would equal the expected utility of the gamble (at point a); i.e, \$4 is the *certainty equivalent* of the gamble.

e. The "risk premium" for Mortimer is \$4 because that is the gap between the certainty equivalent and the expected value of the gamble.

2a. If you think the price of oil will go up by 12% when the interest rate is 10% it would be a mistake to pump today and put your money in the bank. Instead, leave your oil in the ground where it will grow in value at 12% per annum (and if you don't own an oil well borrow at 10% and buy oil).

c. If the expected increase in the price of oil were less than 10% nobody would pump. Only if the expected increase in the price of oil equals the rate of interest will some pump and some not.

Note: The above is based on Hotelling's analysis, which assumed that property rights are secure. If the Swiss bank is safer from revolution than your Saudi oil field, you might want to pump as fast as you can.

3. If the production function is $Q = L^{1/2} + K$, then the marginal product of labor $\partial Q/\partial L = ½ L^{-1/2}$ and the marginal product of capital is $\partial Q/\partial K = 1$. Therefore, the marginal rate of substitution = $½ L^{-1/2}$. Since the MRS does not depend on only the ratio L/Q the production function is *not* homothetic. If a production function is homothetic then the ratio of capital to labor employed by a profit-maximizing firm, the ratio K/L, will be the same regardless of the level of output, given wages and the rental cost of capital.

4. If the BrandX firm has cost function $C(q) = 24 + 4q + q^2$, than ATC = 24/q + 4+ q, AVC = [C(q)-C(0)]/q = 4+q, and marginal cost = dC(q)/dq = 4 + 2q.
If the firm's product were sold in a competitive market at price p = $20, the firm would maximize profit by producing where marginal cost equals price, so we have 4 + 2q = $20 or q = 8. [See graph on next page]

5. If q(p) = 10 – p/3, then p(q) = 30 – 3q is the inverse demand function..
The demand curve goes through points <0,30> and <10,0>

Since revenue is $R(q) = p(q)\, q = 30q - 3q^2$, marginal revenue is dR/dq = 30 – 6q.
The marginal revenue curve goes through points <0,30> and <5,0>.

To maximize profits the firm must produce where marginal revenue equals marginal cost: dR/dq = 30 – 6q = dC/dq = 4 + 2q, or q = 3¼. The maximum price at which this quantity can be sold is p(3¼) = 20¼.

Honors Option: Because dR/dq = (1 – 1/η), if elasticity is less than 1 marginal revenue must be negative. But if marginal revenue is negative, simply selling less will raise revenue without increasing (and probably decreasing) costs because dC/dq ≥ 0.. Therefore, selling less would raise profits.

Moral: A profit maximizing firm will never sell at a point on its demand curve where the elasticity is less than 1.

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The grades, after scaling, ranged from 71 to 116 with an average of 88!
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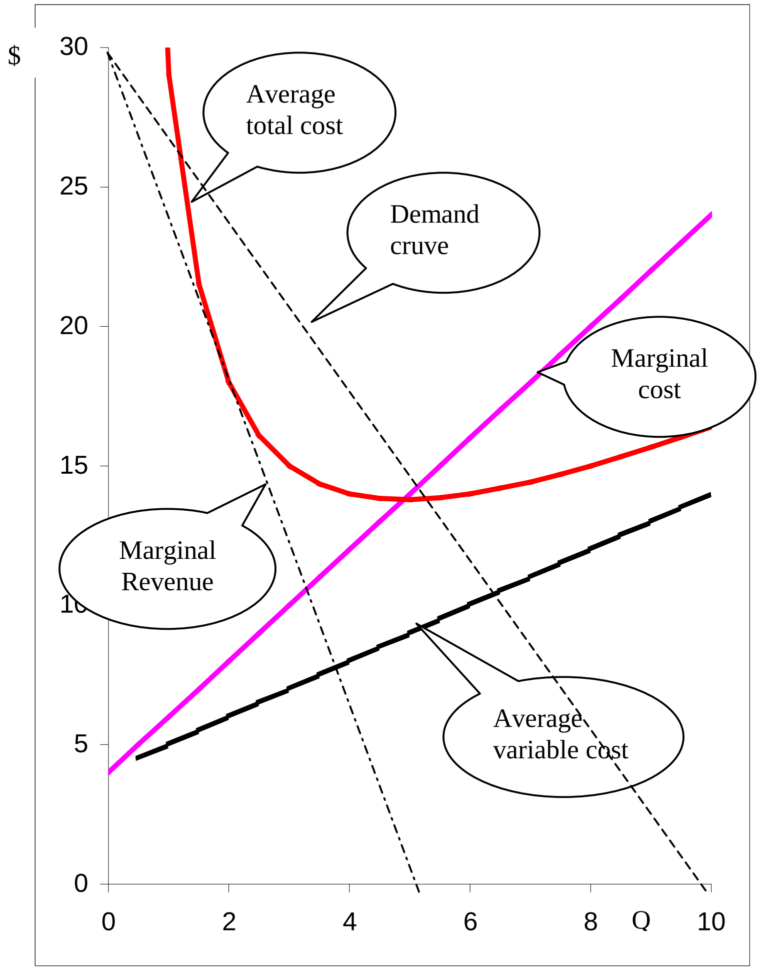
$
30
25
20
15
10
5
0
0
2
4
6
8
Q
10
Average total cost
Demand cruve
Marginal cost
Marginal Revenue
Average variable cost