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PAC 125

Partial Answers, Econ 201/(301?): Final Exam Postmortem

PART 1: IDENTIFICATION: k,g,e,m,b,d,i,c,j,f

PART 2: Please answer five (only 5) of the following seven questions in the space provided

- 2.1 Carefully draw an indifference map showing that a utility maximizing consumer might consume more of a commodity at a higher price. See Varian, page 105
- 2.2 A consumer behaves *as if* she were maximizing the following Lagrangian expression:
$$L(M, X_1, X_2, \lambda) = X_1 - (X_2 + 1)^{-1} - \lambda(p_1 X_1 + p_2 X_2 - M)$$
- Our consumer's utility function is $X_1 - (X_2 + 1)^{-1}$.
 - The marginal utility of X_1 is $\partial U / \partial X_1 = 1$. The marginal utility of X_2 is $\partial U / \partial X_2 = -(x_2 + 1)^{-2}$
 - Derive demand functions $X_1(p_1, p_2, M)$ and $X_2(p_1, p_2, M)$ (You may use either the method of Lagrange or the method of substitution).
$$U(X_2) = M/p_1 - p_2/p_1 X_2 - (X_2 + 1)^{-1}$$
$$dU/dX_2 = -p_2/p_1 + (X_2 + 1)^{-2} = 0 \text{ implies } X_2 + 1 = (p_1/p_2)^{1/2} \text{ or } X_2 = (p_1/p_2)^{1/2} - 1$$
$$\text{Substituting into the income constraint yields } X_1 = M/p_1 + p_2/p_1 - (p_2/p_1)^{1/2}$$

Note that the marginal utility of X_1 is constant.
- 2.3 A monopoly faces demand function $q(p) = 100 - 10p$.
It has total cost function $C(q) = 30 + 2q + q^2/2$.
- The inverse demand function is $p(q) = 10 - q/10$
 - Total revenue is $R(q) = qp(q) = 10q - q^2/10$.
 - Marginal revenue is $dR/dq = 10 - q/5$.
 - $dC/dq = 2 + q = MR = 10 - q/5$ yields $q = 20/3$
 - $\eta = -dq/dp \cdot p/q = 14$
 - What level of output will maximize profits in the long run? Profits are negative in the short run, but above the shut down price. In the long run you go out of business and escape fixed costs by moving to Florida.
- 2.4 The annual net reproduction rate of fish in the Wesleyan Sea is
$$R_t(F_{t-1}) = (2 - F_{t-1})(F_{t-1} - 100,000)/4,000,000 = -.05 + 0.025F_{t-1} - 0.00000025F_{t-1}^2$$

where F_{t-1} is the stock of fish in the sea inherited from the preceding year..
- How many fish will there be in the sea, in stable equilibrium, assuming there are no fisherman? (State of Nature)? $R(F)=0$ has two roots, $F = 2$ and $F = 100000$. Only the largest of these is stable.
 - If fisherman take 250 fish out of the sea each year, the stock of fish will be reduced to the point where $R(F) = 250$, which is a stock of approximately 88,500.
 - To find the maximum sustainable catch, note that $dR/dF = .025 - .0000005F = 0$ yields $F = 50,000$ with $R(F) = 624.5$
 - If the development of radar detectors empowers the fisherman to catch 1,000 fish each year, the stock of fish will crash. Even if each fisherman realizes that the fish face extinction, it will be in the self interest of each fisherman to keep fishing – if I stop fishing the others will still harvest enough to cause extinction. The absence of well-defined property rights leads to market failure.
 - Extinction threatens unless the fisherman organize or the government intervenes to limit the harvest to no larger than the maximum sustainable catch. The use of the new

technology could be prohibited, the number of allowable days of fishing per year could be limited, or a quota on the harvest could be imposed.

2.5. The demand function by the citizens of Dogpatch for physician visits is $q = 1,600 - 8p$, where p is the charge per visit. The citizens wish they had a physician, but no physician will come to Dogpatch unless the town guarantees an annual income of \$100,000 plus \$10 per patient visit.

- Since $R(p) = 1600p - 8p^2$, $dR/dp = 1600 - 16p = 0$ implies that $p = \$100$ will maximize the revenue of a physician practicing in Dogpatch. It will yield total revenue of only \$80,000.
- $(\$200 - \$100) \times 800 / 2 = \$40,000$ of consumer surplus would be generated if physician visits were sold at the revenue-maximizing price.
- The town cannot attract the physician by just charging a fee for service, but when consumer surplus is taken into account it is clear that the consumers would benefit if funds were raised to bring the physician to town. Discriminatory pricing – charging an annual fee plus \$10 per visit – might work. Or the town might use the general tax revenue to help finance the physician. Unless the tax would cause a gross deadweight loss, the gain to the consumers, even after paying the tax, exceeds the cost of the physician. Pricing at marginal cost of \$10 per visit would maximize welfare.

2.6 If $Q_1 = 200 - 2p_1 + p_2$, then $R_1 = 200p_1 - 2p_1^2 + p_2p_1$ and profits are

$$\pi^1 = 200p_1 - 2p_1^2 + p_2p_1 - 20(200 - 2p_1 + p_2)$$

To maximize profits for firm 1 given p_2 , we compute $d\pi/dp_1 = 200 - 4p_1 + p_2 + 40 = 0$ or $p_1 = 60 + p_2/4$.

By symmetry we also have $p_2 = 60 + p_1/4$, which we substitute into the expression for p_1 to obtain $p_1 = 60 + (60 + p_1/4)/4 = 75 + p_1/16$. Therefore, $p_1 = 80$. We then find that $q_1 = 200 - p_1 = 120$, $R = 120 \times 80 = 9,600$, $C = 20 \times 120 = \$2,400$ and profits for firm 1 are $\pi_1 = \$7,200$.

2.7 Two duopolists are each trying to decide whether to price low or price high.

The dominant strategy for Firm #1 is to price low. This yields the most profits, regardless of firm #2's decision. Low pricing also dominates for Firm #2, because that price yields the most profit regardless of what firm #1 does. Low pricing by both firms constitutes the Nash equilibrium. With this pricing strategy neither firm can increase its profits without lowering its profits, given what the other firm is doing.

c. The change in the cost matrix now makes pricing high the dominant strategy for Firm #1. But if Firm #1 prices high, Firm #2 will price low and so Firm #1 will make only \$3,008 – BAD MOVE!

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**HONORS OPTION** (Answer on the back of this page or on a separate sheet of paper.)

Here is a shortcut for answering this question:

Because the price elasticity of demand is 2, the profit maximizing price will always be twice marginal revenue, as can be seen from item 7 of Part I. Since profit maximization requires that marginal revenue equal marginal cost, which is 50, we will always have  $p = \bar{p} = \$100$ .

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The average grade on the final was 88 and the median 89, just a hair above the scores on the quizzes. One student solved the Honors Option, earning a score of 118 on the final.