

Let's play Actuary

The following discussion may help you to complete the Excel Module at the Master Level, but it won't get you through the Actuarial Exams.

The Actuarial Calculation Spreadsheet develops a variety of concepts derived from the survival data in the Lx column of the table. Here is the basic two-step strategy. First figure out the right equation for one of the cells in a column. Then copy/paste that cell into the remainder of the column. If you have it right, you will have filled in the entire column correctly. And you can self-check with the answers in the first and last few rows on the table on the last page of the Module handout.

Columns 9, 10, 11 and 12 use as input the interest rate and either the death benefit or the annuity benefit that are entered in the upper right-hand corner of the spreadsheet. You should not use the numbers 0.05 or 1000 in your spreadsheet; instead, refer to the spreadsheet cell locations. For example, instead of using 0.05 for the interest rate in your calculations, you should refer to cell $\$L\1 (the '\$'s freeze the row and column cell reference). If you follow this procedure, you will be able to see what happens to the annuity and life insurance policy valuations when the interest rate or the benefit changes.

Column 9 gives the actuarial value of a 1 year term life policy. If you were to buy such a policy at age 99, your heirs will definitely get the \$1,000 according to the data in the table, because the probability that you will die by the end of year is 100% ~ Qx is 1.00000 (that's pessimistic, but they had to end the table somewhere).. But the coverage won't be worth that full \$1,000 because the premium is paid up front; the value of the term coverage will be \$952.38 because at 5% that will earn enough interest to be worth \$1,000 at the end of the year; i.e., $E105 * \$L\$2 / (1 + \$L\$1)$; this is $1.0 * \$1000 / 1.05 = 952.38$. So enter $E105 * \$L\$2 / (1 + \$L\$1)$ into cell I105 and verify that you have the right answer for that year. Now Copy/Paste it down into the rest of the column and verify the results. Verify that the value of the policies drop if you raise the interest rate. Insurance companies do well when interest rates are high!

The one-year annuity is a bit more involved. A one-year annuity is not worth anything if you are 99, because the table says that no one lives to be 100. At age 98, Px in column 6 is 0.27533, which says that you have a 27.5% probability of living long enough to get the \$1,000. But you have to wait a year, so the interest factor means that the valuation should be $0.27533 * \$1000 / 1.05$. If you key in the equation correctly, you will get the right answer. After copy/paste you will have the whole column.

A single payment life policy is harder to evaluate. The best way to conceptualize the problem is to start at age 99 and work backwards recursively. If you are 99 years old, it's the same as one-year term life. If you buy it at age 98, you are getting two things simultaneously: (1) you are getting both a single payment life because you may die at age 98, which is worth \$690.16; (2) you are also getting a one-year deferred SP Life of someone of age 99. The deferred single payment life is not worth the full \$952.38 for two reasons – you may die during your 98th year so it's never relevant. And if you do survive to age 99, which has a probability of 0.27533, the payout is deferred for a year, which means that your premium has a year to earn interest. So the deferred single payment life is worth $0.27533 * \$952.38 / 1.05$. That plus the \$690.16 gives you the value of the SPLife policy purchased at age 98. Key that into the K104 appropriately, copy/paste it down the column, and you should be all set!

The same *backward recursion* strategy also works with an annuity. Figure out the equation for the value in year 97 by noting that it is the sum of a one-year annuity plus a deferred life annuity (which is discounted because it is purchased in advance and you get it only if you survive into your 98th year). It's tricky, but once you get it right for 97 you can copy it all the way back to year one. Tell your rich uncle that this is what you want for a graduation present.

Mike Lovell, March 1, 1999

