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# Module #6: Mathematica

| | TOPIC | Due Dates: Basic | Advanced | Master |
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| #6 | Mathematica | April 16 | April 23 | April 23 |

Before attempting this exercise you must carefully study "Working with Mathematica."

Do the following calculations, using Mathematica. Do not submit your results by Email. Instead print out the notebook, or better yet, copy and paste relevant parts of the notebook into a word processing document.

## I. BASIC LEVEL:

1. Solve the following problems:
   a. How much will your $1,000 inheritance be worth after 20 years if you place it in the bank earning interest at 5% per annum?. Hint: $A = P(1+r)^t$
   b. A construction project costing $1,000 will yield an apartment house that can be sold for $2,000 after three years. Solve for the "internal rate of return."
   c. Hints: Find r in the expression $\$2,000 = \$1,000(1+r)^3$; i.e., $1000(1+r)^3 - 2000 = 0$. You may have to use N[Solve] to get a rounded off (numerical) answer.

2. Solve for the roots of the polynomial $x^2 + 4x + 2$

   a. Graph the polynomial $x^2 + 4x + 2$ over the range $0 \leq x \leq 10$ of the real number line
   b. Now solve for the general solution to quadratic equations ( $ax^2 + bx + c$)
   c. Take the first and second derivatives of $x^2 + 4x + 2$
   d. Take the first and second derivatives of $ax^2 + bx + c$ with respect to x

## II. ADVANCED LEVEL:

1. Here are four alternative Production Functions (microeconomics), where Q = units of output, K = units of capital input, and L = units of labor input, with constants a > 0, b > 0:

   Perfect substitutes: $Q = aK + bL$
   Leontief: $Q = \min[aK,bL]$
   Cobb-Douglas: $Q = K^a L^b$
   CES $Q = [aK^b + (1-a)L^b]^{1/b}$

   Choose values for a and b (e.g., a = 1/4; b = ¾). Then, using ContourPlot Mathematica functions, show what the isoquants and the production function look like for each of the production function over some range of nonnegative values for K and L.

   Also use Plot3D for the Cobb-Douglas production function

2. A simple four equation macroeconomic application (an IS curve):

$$Y = C + Inv + G + X - M$$
$$C = 100 + 0.85\ Y$$
$$Inv = \max[50 - 100r, 0]$$
$$M = 0.10\ Y$$

Here Y = national income, C = consumption, Inv = investment, M = imports, X = exports, G = government spending, and r = the interest rate

a. Solve for the equilibrium values of Y, C, Inv, and M if X = 30, G = 50, and r = 10%.
b. Determine how the equilibrium values will change if G increases to 60.
c. Determine how the equilibrium values will change if G= 60 and r = 20%.

3. Suppose that the probability that a part will fail by time t is given by the function $F(t) = 1 - e^{-rt}$ for $t \geq 0$. This is the exponential distribution. It is the simplest form of duration model, which can be used to analyze the length of spells of unemployment, the time until the next stock market bust, etc.

a. For r = 0.2, Plot F(t) over the range $0 < t < 20$
b. Solve for the median time of part failure, given r = 0.2 Hint: $F(t_{median}) = ½$.
c. Repeat a and b for r = 0.4. *Bonus:* Explain analytically the relationship between $t_{median}$ and the parameter r.
d. Use Mathematica to solve for the density function f(t) = dF(t)/dt. for r = 0.2; and again for r = 0.4%.
e. Plot the density function for r = 0.2.

## III. MASTER:

1. You are to compute the mean of the exponential distribution for r = 0.2 and again for r = 0.4. This requires that you compute $E(t) = \int_0^\infty t f(t)dt$, where f(t) is given by 3d above. In general, for any r > 0, determine E(t) as a function of r.

2. The equation for the normal density function is $f(x) = \frac{1}{\sqrt{2ps^2}} e^{-\frac{1}{2}(\frac{x-m}{s})^2}$ (see W&W, equation 4-=26, p 132).
   a Plot the normal density function.
   b. Solve for the cumulative density function $F(x) = \int_{-\infty}^{x} f(x)dx = P(X \leq x)$ for the standardized normal case of $\mu = 0$ and $\sigma = 1$. Compare 1-F(2) with the value in W&W..
   c. Solve for $F(\infty)$. Why must this integral equal one?

BONUS: Plot the IS relationship you found in question II.2, macro economics. (The IS curve is traditionally plotted with Y on the abscissa and r on the ordinate, given G and X).