Problem Set #9: GDP Accounting and the Multiplier

Due 2:00, November 17, 2000

Purpose: After a brief look at national income accounting, this problem set presents the simple multiplier model explanation of how fiscal policy (changes in government spending and tax policy) may influence the pace of economic activity and unemployment. This is a grossly oversimplified model which is valid, at best, only when there is ample excess capacity. Later we shall be elaborating on this model in order to build a framework for the study of inflation and monetary policy — be patient!

Historical Note: The mode of analysis of this handout was developed during the great depression by R. F. Kahn, "The Relation of Home Investment to Unemployment" Economic Journal, June 1931. It was a central feature of Keynes Theory of Employment, Interest and Money, 1937.

1. National Income Accounting: The following data are for Simpleland, a mythical country similar in many respects to the United States, but somewhat simpler:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>I Gross private investment</td>
<td>$45</td>
</tr>
<tr>
<td>G Government spending on goods and</td>
<td>$80</td>
</tr>
<tr>
<td>services</td>
<td></td>
</tr>
<tr>
<td>C Consumption spending</td>
<td>$250</td>
</tr>
<tr>
<td>D Capital Consumption Allowances</td>
<td>$20</td>
</tr>
<tr>
<td>(depreciation expense)</td>
<td></td>
</tr>
<tr>
<td>X Exports</td>
<td>$10</td>
</tr>
<tr>
<td>M Imports</td>
<td>$15</td>
</tr>
<tr>
<td>Tp Personal Income Tax</td>
<td>$35</td>
</tr>
<tr>
<td>CP Corporate Profits (before tax)</td>
<td>$50</td>
</tr>
<tr>
<td>Div Dividends</td>
<td>$23</td>
</tr>
<tr>
<td>Tc Corporate Profits Tax</td>
<td>$18</td>
</tr>
<tr>
<td>Ti Indirect Business Taxes</td>
<td>$10</td>
</tr>
<tr>
<td>Ts Social Security Taxes</td>
<td>$13</td>
</tr>
<tr>
<td>Tr Government Transfer Payments</td>
<td>$16</td>
</tr>
</tbody>
</table>

   a. Calculate GDP:
      \[ Y = C + I + G + X - M \]
      \[ Y = 250 + ____ + 80 + 10 - 15 = 370 \]

   b. Calculate Net National Product:
      \[ NNP = Y - D = ____ - ____ = ____ \]

   c. Calculate Corporate Profits After Tax:
      \[ CP_{at} = CP - Tc = ____ - ____ = ____ \]

   d. Calculate Corporate Retained Earnings (Undistributed Profits):
      \[ CRE = CP_{at} - Div = ____ - ____ = ____ \]

   e. Calculate total tax revenue:
      \[ T = ____ + ____ + ____ + ____ \]

   f. Calculate the Government Deficit:
      \[ GDeficit = G + Tr - T = ____ + ____ - ____ = ____ \]

   f. Calculate Disposable Income:
      \[ Yd = Y - D - T + Tr - CRE = ____ - ____ - ____ + ____ - ____ = ____ \]

      [The above equation is equivalent but not identical to that in the text.]

2. Empirical Evidence on Okun's Law: Okun's law is the proposition that every 1% excess of unemployment over the full employment level causes GDP to drop by three percent below its potential. In his original presentation, Okun used a factor of 3.0 in his famous equation. While the basic form of the equation has stood the test of time much better than anyone, including Okun, had any right to expect, many economists think that a smaller coefficient, perhaps 2.0, is more appropriate for the 1990's.

   A preliminary statistical analysis suggests\(^1\) (t denotes the t-th quarter (3 month period)):
   \[ Y_t - Y_{t-1}/Y_{t-1} = 0.0081 - 1.83 \cdot (U\%_t - U\%_{t-1}) + e. \]
   The e on the extreme right of the equation represents a random error, indicating that the equation does not fit the data perfectly; for this exercise you can assume e = 0. The equation implies that output must grow by 0.81% each quarter just to keep unemployment constant (because of population growth and increased worker productivity). It also implies that to reduce the unemployment rate by 1% at time t it is necessary for GDP to grow by an additional 1.83%.

   In 1982, when the civilian unemployment rate averaged 9.7%, GDP was $4,624 billion (1992 dollars).

   Construct the best estimate you can, using the above evidence, of how much more output (1992 $) could have been produced if the unemployment rate had been reduced to 5.5%.

3. A Simple Macroeconomic Model:\(^2\) Suppose that in Simpleland the GDP Identity holds:

   \[ Y = C + I + G + X - M \]

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\(^1\) The equation was estimated by the method of “least squares,” which is explained in Econ 270: Quantitative Methods.
(1) \[ Y = C + I + G + X - M \]

Also,

(2) \[ Y_d = Y - T - CRE - D + T_r \]

Suppose corporate retained earnings are 5% of GDP, transfer payments are \( T_r = 5 \), depreciation is 10% of GDP and taxes are

(3) \[ T = -25 + .25 Y \]

Further, consumption is

(4) \[ C = 10 + 2/3 Y_d \]

**Definition:** \( dC/dY_d \), the slope of the plot of \( C \) on \( Y_d \) is called the "Marginal Propensity to Consume out of Disposable Income" (MPC\(_{yd}\)). For the above example, \( dC/dY_d = 2/3 \).

a. Complete the following table

<table>
<thead>
<tr>
<th>Y</th>
<th>T</th>
<th>CRE</th>
<th>D</th>
<th>Yd</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>100</td>
<td>25</td>
<td></td>
<td></td>
<td>230</td>
</tr>
<tr>
<td>1000</td>
<td>225</td>
<td></td>
<td>100</td>
<td>630</td>
<td>430</td>
</tr>
<tr>
<td>2500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Draw a neat graph showing how disposable income and consumption spending are influenced by \( Y \). [Hint: Plot \( Y \) on the horizontal axis (abscissa) and both \( Y_d \) and \( C \) on the vertical axis (ordinate)].

The slope of your Con\( Y \) line is _____ (i.e., each $1.00 increase in GDP will lead to a rise of $_____ in consumption);

c. Determine from the above information the parameters (coefficients) of the following equations:

\[ Y_d = d_0 + d_1 Y \] (disposable income function)
\[ C = c_0 + c_1 Y \] (consumption as a function of GDP

**Definition:** The slope of the Con\( Y \) line, \( dC/dY \), is the "Marginal Propensity to Consume out of GDP" (MPC\(_y\)).

d. Explain why \( MPC_y \neq MPC_{yd} \)? (Hint: Consider footnote 3)

4. Suppose that in Simpleland the exogenous variables have the values \( G = \$300 \), \( I = \$200 \), \( X = \$100 \) and \( M = \$30 \). Then autonomous spending \( (G + I + X - M) \) totals $______.

Solve for the following endogenous variables:

\[ Y = \underline{________} \quad T = \underline{________} \quad Y_d = \underline{________} \quad G - T = \underline{________} \quad C = \underline{________} \]

**Definitions:** The "endogenous" variables are the unknowns to be solved for on the basis of information about the model's "parameters" (coefficients) and the specified values of the "exogenous variables".

[Self-Test: Your solution must yield a value of \( C \) that is on the consumption function (equation 4); also, taxes must be given by equation (3); equations (1) and (2) must also be satisfied]

5. Consider the following simplified model involving 3 endogenous variables:

(1) \[ C = c_0 + c_1 Y_d \] (Consumption Function)

(2) \[ Y_d = d_0 + d_1 Y \] (Disposable Income Function)

(3) \[ Y = C + I + G + X - M \] (GDP identity)

For Simpleland, the parameters of equation (1) can be taken to be those specified in equation 4 of question 3. The appropriate values for the disposable income equation are those you solved for in question 3c.

a. Use these parameter values to determine the equation relating consumption spending to GDP in Simpleland.

b. Find the Reduced Form Equation showing the endogenous variable \( Y \) as a function of exogenous variables \( G, I, X \) and \( M \) and the parameters of the model.

**Definition:** A Reduced Form Equation is an equation determining one of the endogenous variables in terms of the exogenous variables. (There should be one reduced form equation for each endogenous variable)

c. Determine \( \Delta Y/\Delta G \), the government spending multiplier, as a function of the parameters of this model. [HINT: \( \Delta \) denotes a finite change. Differentiate the Reduced Form Equation — the model is linear, so the change in GDP is proportional to the change in government spending].

d. Explain why the multiplier changes if the model is augmented by converting the exogenous variable \( M \) (imports) into an endogenous variable by adding the equation \( M = m_0 + m_1 Y \). (\( m_1 \) is the marginal propensity to import)

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\(^2\) In many elementary textbooks, the derivation of the multiplier is simplified by neglecting foreign trade, complications and the gap between GDP and disposable income (i.e., \( Y = Y_d \) and \( M = X \)). This is too simple for Econ 105.