Abstract

Under what circumstances will a monopolistic competitive market be characterized by too little or too much quality variation? How do excise taxes affect the monopolistic competitive equilibrium? Can an increase in the minimum wage lead firms to employ more workers but cause a reduction in industry-wide employment? What will be the multiplier effect of a balanced-budget increase in government taxation and spending? These are among the questions to be analyzed in this paper. The analytical framework is developed by first specifying certain properties that the demand function for commodities in a monopolistically competitive market may conveniently be expected to satisfy and then determining the utility function that will generate it.

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* I am indebted to John Bonin, Gilbert Skillman and Gary Yohe for helpful comments on an earlier draft of this paper.
1. Introduction

The essential features of Edward Chamberlin's theory of monopolistic competition are conveniently summarized using his ingeniously constructed graph. Chamberlin made his analysis tractable by assuming that the demand for each firm's product depends only on its own price and the average price being charged by the other firms in the industry. On his graph he plotted two types of demand curves for the representative firm: First, there is the family of \( dd' \) demand curves, each of which is plotted for a specific average price being charged by all the other firms in the industry — three of these \( dd' \) curves are plotted on Figure 1. They are downward sloping because of product differentiation. The other type of demand curve plotted on the graph, \( DD' \), shows how much the firm would be able to sell if all the other firms in the industry were also to charge whatever price it is charging. Also plotted on the graph is the average total cost curve.

Tangency point \( e \) represents the long-run equilibrium solution, given Chamberlin's "large group case" assumption that there are enough firms in the industry to make it reasonable for each firm to assume that other firms will not respond to adjustments in its own price.\(^1\) Because of free entry and exit, economic profit is zero at point \( e \) (average cost equals price). Further, \( e \) is a profit maximizing point because at any other price the demand curve is below the average total cost curve, implying losses instead of profits.

Two basic strategies have been followed in constructing models of monopolistic competition. One strategy, building on the pioneering work of Hotelling [1928], is to use a spatial rep-

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\(^1\) The number of firms required for the validity of this large-group assumption of zero conjectural variation will be reduced if there is a reasonable range of limited rationality. See the discussion of limiting distributions in Stiglitz [1986], p 56 and footnote 63.
presentation: Thus Lovell [1970] assumed that the differences in consumer preferences were uniformly distributed along a beach or road that was infinitely long while Salop [1979] assumed they were uniformly distributed along a beach circumventing an island or the shore of a lake – these are alternative procedures for avoiding endpoint complications. Another strategy, followed by Dixit and Stiglitz [1977], is to assume that the preferences of consumers are captured by a single utility function in n commodities, where n itself is determined by market conditions. They invoked a CES style utility function and derived demand curves from it.

In the second section of this paper I develop a strategy that is similar but distinct from that of Dixit-Stiglitz. My approach is to list nine properties that it might be reasonable to assume that demand curves in a monopolistic competitive market would possess. Then I derive the class of demand functions satisfying these nine properties. Next I specify the corresponding utility function that would generate this class of demand functions. After developing this groundwork I go on to determine the properties of the monopolistic competitive Nash equilibrium, given the number of firms in the industry. Then I explore the long run zero profit equilibrium when there is free entry into the industry.

The question of whether monopolistic competitive models generate too much or too little product diversity is explored in Section 3 of this paper. This issue has been examined by a number of investigators, including Lovell [1970], Spence [1976], Dixit-Stiglitz [1977], Sattinger [1983], and Stiglitz [1986]. Within the context of the model developed in this paper, I demonstrate that there is likely to be too much diversity if industry sales have a quite inelastic response to an increase in product variety and too little if sales are more responsive. Section 4 examines the incidence of license fees and excise taxes on the monopolistic competitive equilibrium. I show how a subsidy might be used to establish the optimal number of firms in the industry.

Section 5 shows that an increases in the minimum wage will lead to increased employment in each industry but a decline in the number of firms, thus reconciling the increased size of firm effect reported in the Card and Krueger [1995] “event study” with the conventional view that the minimum wage causes unemployment.

Section 6 develops a simple general equilibrium framework based on monopolistic competition. I show within the context of this multiple equilibrium model that an increase in government expenditure financed by taxes will lead to a balanced budget multiplier, just as predicted by Keynesian analysis. I also explore within the general equilibrium framework the effects on the level of economic activity and employment of a tax policy designed to achieve the optimal number of firms in the industry.

2. The Model

2.1. The Demand Function

Under monopolistic competition, the quantity that the ith firm will sell depends not only on its own price but also on the prices charged by the other n-1 firms in the industry:

\[ q_i(p_1, \ldots, p_n) \]  

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2 The linear beach model of Hotelling may be most appropriate for considering the location of retail outlets along an El Camino Real, while the circular model used by Vickrey and Salop may be particularly appropriate in considering the color spectrum, as with paints or fabrics.
In considering this demand relationship it proves useful to enumerate various properties that such a function might reasonably be expected to satisfy. Let us begin by following Chamberlin in assuming that what matters is only the average price charged by the other firms in the industry, regardless of the variance or skewness of the distribution of prices. That is to say, assume that \( q_i(p_1, \ldots, p_n) \) is of the restricted class for which there exists a function \( q_i(p_i, \bar{p}_i, n) \), such that

\[
q_i(p_i, \bar{p}_i, n) = q_i(p_1, \ldots, p_n)
\]

for all \( p_i \) and \( n \),

where \( \bar{p}_i = \frac{\sum p_j}{n-1} \) is the average price being charged by the other \( n-1 \) firms of the industry.

This is the first of nine properties that an appropriately specified demand function might reasonably be expected to satisfy:

1. There exists a function \( q_i(p_i, \bar{p}_i, n) \) such that \( q_i(p_i, \bar{p}_i, n) = q_i(p_1, \ldots, p_n) \) for all \( p_i \) and \( n \). Chamberlin’s average simplification

2. \( q_i(p, \bar{p}, n) = q_j(p, \bar{p}, n) \) for all \( p, p_i, n, i \) and \( j \). Chamberlin’s symmetry simplification

3. \( \frac{\partial q_i}{\partial p_i} < 0 \) The Law of Demand

4. \( \frac{\partial q_i}{\partial \bar{p}_i} > 0 \) An increase in the average price charged by competitors will increase the \( i \)th firm’s sales if it does not change its own price.

5. \( q_i(p, \rho \bar{p}, n) < q_i(p, \bar{p}, n) \) for any \( \rho > 1 \) Industry sales will decline if all firms raise their prices proportionately.

6. \( q_i(p_i, \bar{p}_i, n) = 0 \) implies \( q_i(p_i, \bar{p}_i, n+1) = 0 \) If a firm is priced out of the market, it will still be priced out of the market if a new firm charging the same price enters the industry.

7. If \( q(p_{n+1}, \bar{p}, n+1) > 0 \) then \( 0 < q_i(p_i, \bar{p}_i, n+1) < q_i(p_i, \bar{p}_i, n) \) The entry of a new firm with positive sales decreases but does not eliminate the sales of existing firms, given prices.

8. \( q_{n+1}(p_{n+1}, \bar{p}_{n+1}, n+1) > 0 \) implies \( \sum_{i=1}^{n+1} q_i(p_i, \bar{p}_i, n+1) > \sum_{i=1}^{n} q_i(p_i, \bar{p}_i, n) \) The entry of a new firm with positive sales will increase total industry sales, given prices.

9. \( q_i(p_i, \bar{p}_i, n) \) is linear in \( p_i \) and \( \bar{p}_i \) (apart from sign & income constraints). Analytical simplicity (Occam’s Razor).

While six of these properties are eminently reasonable, #1, #2 and #9 are motivated by a desire to keep the model analytically tractable. The objective is to make the resulting model as simple as possible, but no simpler.
Theorem 1:
Any demand function satisfying these nine properties can be expressed as
\[ q_i(p_i, \bar{p_i}, n) = d_0(d_1 - p_i + d_2 \bar{p_i})/c(n) \] (3)
with inverse
\[ p_i(q_i, \bar{p_i}, n) = d_1 + d_2 \bar{p_i} - q_i c(n)/d_0, \] (4)
where \( c(n) \) is an appropriately specified index of the intensity of competition, such as \( c(n) = \sqrt{n} \) and \( d_0 > 0, d_1 > 0, 0 < d_2 < 1 \).

Proof in Appendix.

Before proceeding, we must obviously verify that demand function (3) is compatible with the assumption that the representative consumer is a utility maximizer. As explained more precisely in the Appendix, Theorem 2, a maximizing consumer with utility function
\[ U(q_1, \ldots, q_n, Y) = u_0 + u_1 \sum q_i - u_2 \sum q_i^2/2 + u_3 \sum \sum q_i q_j + Y, \] (5)
where \( Y \) is a numéraire good with constant marginal utility, will have a demand function for the \( i \)th good of the form (3), unless income is very low.\(^3\) The Appendix explains how the \( u_i \) parameters in equation (5) depend on \( n \).

To find Chamberlin’s DD’ demand curve (cf. Figure 2) generated by (3), note that if \( p_i = \bar{p_i} \), demand for the output of each firm will be
\[ q_i(\bar{p_i}, n) = d_0(d_1 - d_2 \bar{p_i})/c(n) \] (6)
with inverse demand function
\[ \bar{p_i}(q_i, n) = d_1/(1-d_2) - q_i c(n)/[d_0(1-d_2)]. \] (7)

The market (industry) demand curve implied by (3) is
\[ Q(p_i, n) \equiv \sum q_i(p_i, \bar{p_i}, n) = nd_0(d_1 - (1-d_2) \bar{p_i})/c(n); \] (8)
only the first moment \( \bar{p_i} \) of prices and the number of firms \( n \) matter in determining industry sales. The inverse market demand curve is
\[ \bar{p_i}(Q, n) = d_1/(1-d_2) - Qc(n)/[nd_0(1-d_2)]. \] (9)
As an interesting special case, if \( c(n) = n^{1/\gamma} \), \( 0 < \gamma < 1 \) the elasticity of market demand with respect to the number of firms will be \( \gamma \) and
\[ Q(\bar{p}, n) = d_0(d_1 - (1-d_2) \bar{p}) n^{\gamma}. \] (10)

2.2. Maximizing Profit, Given \( \bar{p_i} \) and \( n \)

It is useful to keep things as simple as possible by working with a linear total cost function\(^4\)

\(^3\) To facilitate the analysis, it will be assumed throughout that internal maximums are obtained. This utility function is closely related to one considered by Lovell [1970, p 123].
\[ C(q_i) = k_0 + k_i q_i, \quad k_i > 0. \]  
(11)

This may be a reasonable approximation of the costs of producing a book, where \( k_0 \) denotes the sum of the value of the author’s time, typesetting and proofreading costs and so forth while \( k_i \) is the per copy cost of paper, ink, binding, etc. It may also be a useful first approximation of the costs of producing computer software and pharmaceuticals, where \( k_0 \) is development cost and \( k_i \) production costs.

From (4) revenue is
\[ R(q_i, \bar{p}_i, n) = d_1 q_i + d_2 \bar{p}_i q_i - q_i^2 c(n)/d_0; \]  
(12)

therefore, profit will be
\[ \pi(q_i, \bar{p}_i, n) = R(q_i, \bar{p}_i, n) - C(q) = \]
\[ = d_1 q_i + d_2 \bar{p}_i q_i - q_i^2 c(n)/d_0 - k_i q_i \]
\[ = -k_0 + (d_1 + d_2 \bar{p}_i - k_i) q_i - q_i^2 c(n)/d_0. \]  
(13)

A necessary condition for profit maximization, given \( \bar{p}_i \), is that
\[ \frac{\partial \pi}{\partial q_i} = d_1 + d_2 \bar{p}_i - k_i - 2q_i c(n)/d_0 = 0. \]  
(14)

This condition yields as the profit maximizing level of output, given \( n \) and \( \bar{p}_i \),
\[ q^*_i = (d_1 + d_2 \bar{p}_i - k_i)d_0/2c(n). \]  
(15)

Substituting into (4) reveals that the best price to charge, given \( \bar{p}_i \), is
\[ p^*_i = (d_1 + d_2 \bar{p}_i + k_i)/2. \]  
(16)

The firm should shutdown if this price is less than average variable cost; i.e., \( p^*_i < k_i \), or from (16), \( d_1 + d_2 \bar{p}_i < k_i \). Neither the profit-maximizing price nor the shutdown price depend on \( n \), the number of firms in our industry.

2.3. Nash Equilibrium, given \( n \).

In a Nash Equilibrium, no firm can increase its profits, given the price charged by the other firms in the industry;\(^5\) with symmetry, \( p_i = p^*_i = \bar{p}_i \), which from (16) implies that \((2-d_2)p_i = d_1 + k_i\) or
\[ p^N_i = (d_1 + k_i)/(2-d_2) \]  
for all \( i \).

Thus we have the surprising result that any demand function satisfying properties 1 through 9, coupled with constant marginal cost, implies that the short-run market equilibrium price is independent of the number of firms in the industry!\(^6\) On substitution into (15):

\(^4\) Lovell [1970], Dixit and Stiglitz [1977], Salop [1979] and many other investigators of monopolistic competition have used this cost function. Lovell [2000] uses a more elaborate cost function in which optimizing inventory policy leads to declining marginal cost.

\(^5\) Spence [1976 (June), p 235] works with quantity-Nash rather than price-Nash equilibria (Cournot rather than Bertrand); i.e., he assumes that firms believe that their rivals will adjust price so as to keep sales volume fixed. He defends his assumption with the argument that it is analytically tractable, that the general results are not sensitive to this assumption, and the belief that it captures a part of the tacit coordination to avoid all-out price competition that he believes characterizes most industries. Lovell [1970] considered a market territory protecting price policy in which it was assumed that rival firms would lower price by however much was necessary in order to preserve their market territory.
\[
q_i^N = d_0(d_1 - (1-d_2)k_1) / (c(n)(2-d_2)).
\] (18)

Profits of a maximizing firm in Nash equilibrium are
\[
\pi_i^N = q_i^N(p_i^N,k_i) - k_0 =
\]
\[
d_0(d_1 - (1-d_2)k_1)^2 / (c(n)(2-d_2)^2 - k_0). \] (20)

### 2.4. Free entry and Exit

When there is free entry, economic profits will be driven to zero in the long run. One might hope that the price would be lower in the long run when entry is free rather than restricted. The model considered here shows that this need not happen. Because \( n \) does not enter into equation (17), an increase in the number of firms does not influence price, once there are enough firms to lead to the establishment of the zero profit equilibrium. This does not mean that consumers have nothing to gain if more firms enter the industry, but the gain will be in the form of greater product variety rather than a reduction in price.

It is useful to know how many firms will be in the industry when market forces succeed in establishing the Nash equilibrium price. When \( \pi = 0 \), equation (20) implies
\[
c(n) = d_0(d_1 - (1-d_2)k_1)^2 / k_0(2-d_2)^2. \] (21)

If we are also willing to specify \( c(n) = n^{1-\gamma}, 0 < \gamma < 1 \), then we have
\[
n^e = \{d_0(d_1 - (1-d_2)k_1)^2 / k_0(2-d_2)^2\}^{1/1-\gamma}. \] (22)

If the economy is observed in a Nash equilibrium, a simpler strategy may be used to determine the long run characteristics of the model. In any Nash equilibrium the profits of the representative firm are \( \pi^N = (p_i^N - k_1)q_i(p_i^N,p_i^N,n^N) - k_0 \) while in long-run equilibrium \( \pi^e = (p_i^e - k_1)q_i(p_i^e,p_i^e,n^e) - k_0 = 0 \), thanks to free entry and exit; therefore,
\[
\frac{\pi^N + k_0}{k_0} = \frac{q_i(p_i^N,p_i^N,n^N)}{q_i(p_i^N,p_i^N,n^e)} = \frac{c(n^e)}{c(n^N)}, \] (23)

or
\[
n^e = c^{-1}\left[ c(n^N) \left( \frac{\pi^N + k_0}{k_0} \right) \right]. \] (24)

For example, if \( c(n) = n^{1-\gamma}, 0 < \gamma < 1 \), then \( n^e = n^N \left( \frac{\pi^N + k_0}{k_0} \right)^{1/1-\gamma} \). Also, since \( p_i^e = p_i^N \) and \( \pi_i = 0 \), we will have \( (p_i^N - k_1)q_i^N - k_0 = 0 \) as well as \( (p_i^N - k_1)q_i^N - k_0 = \pi^N \). Therefore, the size of the representative firm in the long run must be
\[
q_i^e = q_i^N \left( \frac{k_0}{\pi^N + k_0} \right). \] (25)

The price that prevails in the long run need not be the Nash equilibrium price. Market forces may be frustrated by government regulation, such as minimum pricing restrictions. Also, retail price maintenance or fair pricing codes imposed by trade or professional associations may

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6 This result certainly does not hold only for the linear demand function utilized in this paper. In particular, the same result holds if the demand curves have constant elasticity; i.e., linear in the logs rather than linear.
also succeed in holding price above its Nash equilibrium level of equation (17). Not too many years ago it was illegal to retail furniture and appliances in Connecticut at less than wholesale plus a 6% markup. Also, prior to the 1973 court ruling in the case of Goldfarb vs Virginia Bar that the practice violated federal antitrust statutes, county or state bar associations enforced minimum fee schedules almost everywhere in the United States. While the price that prevails will influence the number of firms in the industry, unless free entry is prevented economic profit will be driven to zero in the long run. How many firms will survive in the long run if they are all charging the same price $p$? With zero profits we have, thanks to equation (6),

$$q(\bar{p}, n) = \frac{k_0}{(\bar{p} - k_1)} = \frac{d_0(d_1 - (1-d_2) \bar{p})}{c(n)}.$$  

Therefore,

$$c(n) = d_0(d_1 - (1-d_2) \bar{p})/(\bar{p} - k_1)/k_0$$

If the elasticity of industry demand with respect to product variety is constant, $c(n) = n^{1-\gamma}$ and

$$n = \left\{d_0(d_1 - (1-d_2) \bar{p})/(\bar{p} - k_1)/k_0\right\}^{1/(1-\gamma)}.$$  

2.5. Numerical Example

It proves helpful to consider the hypothetical numerical example presented on Table 1, which is calculated with $c(n) = n^{1/2}$ and the following hypothetical parameter values: $d_0 = 10$, $d_1 = 11$, $d_2 = 0.75$, $k_0 = 64$, and $k_1 = 4$. The demand curve is

$$q = 10(11 - p_i + 0.75 p_0)/n^{1/2}$$

and the total cost function

$$C = 64 + 4q.$$  

Each row of the table is generated for a particular combination of $p_i$ and $\bar{p}_i$.

The ith firm would prefer to have all other firms price themselves out of the market by charging at least $44$ – line 15 reveals that if every firm were to charge this price, industry sales would be zero. If other firms were to charge $44$, line 20 tells us that the ith firm could maximize its own (monopoly) profits by cutting its price to $24$. This obviously will not come to pass because all the other firms have the same incentive to charge less. At the opposite extreme, line 16 shows that marginal cost pricing cannot prevail in the industry: a price of $p = 4.00$ cannot prevail because when all other firms are charging that price, each firm has an incentive to raise its own price to $p_i = 9.00$.

Lines 1, 3, 7, 12, 15, 17, and 21 represent points on the DD’ curve (i.e., $p_i = \bar{p}_i$). Only at one price, $\bar{p} = 12$, will each firm maximize its own profits, given what the other firms are doing, by charging that same price – this is the market's Nash equilibrium on line 17. But with only 25 firms in the industry each firm will sell 16 units and enjoy positive profit of $64$, as may be seen from the table. Since positive profits provide an incentive for new firms to enter, the industry cannot be in long-run monopolistic-competitive equilibrium.

To find the long-run equilibrium, we recall that the price will remain at $12$. Using equation (24), with $c(n) = n^{1-\gamma}$, we find that in equilibrium the number of firms will be $n^e = 100$.  

It is also interesting to ask what price would result from merger or collusion. In general, the joint-profit-maximizing price, which might result from price leadership, collusion, buyout or cartel, is

\[ p^* = d_i/(2(1-d_2)) + k_i/2. \tag{29} \]

The collusion price, like the equilibrium monopolistic competitive price, does not depend on the number of firms in the industry, although the feasibility of maintaining price discipline is obviously less when there are more firms. As indicated by Row 21 on Table 1 and Table 2, for this

<table>
<thead>
<tr>
<th>Number of firms</th>
<th>Demand Function Parameters</th>
<th>Cost parameters:</th>
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<td>Profit Maximizing ( p_i ), given ( p )</td>
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particular example, the joint-profit maximizing price at $24 is just twice the price of $12 that will prevail under monopolistic competition.

3. Optimal diversity

Do market forces yield too little or too much product diversity? Debate on this issue opened in the 1930’s with the simultaneous publication of two economic classics. In her *Imperfect Competition* Joan Robinson [1933] pointed out that the fact that the demand curve facing the typical
firm is downward sloping, thanks to product differentiation, means that the same industry output could be produced at lower resource cost by limiting the number of firms in the industry. In his *Monopolistic Competition* [1933], Edward Chamberlin argued that the additional cost is well spent because the greater product variety leads to greater consumer satisfaction. In exploring this issue it is necessary to seek a balance between the increase in consumer surplus occasioned by the greater product variety against the additional resource costs required for their production. Lovell [1970] demonstrated within the context of a spatial model that the monopolistically competitive equilibrium might be characterized by the wrong number of firms. Dixit-Stiglitz [1977, p 308] concluded that “there was some presumption that the market solution would be characterized by too few firms…” Working with a quite different model, Spence concluded [1977, p 234] that “high own price elasticities and high cross elasticities create an environment in which monopolistic competition is likely to generate too many products. Conversely, low own-price elasticities and low cross elasticities constitute an environment in which product diversity will be too low.” Sattinger [1983] has shown within the context of a spatial model that over entry is more likely if the distribution of consumer preferences, is highly skewed; e.g., Pareto.

In investigating this issue within the context of the model developed in this paper, it is best to adopt the standard assumption that the objective is to maximize the sum of profits plus consumer surplus; or equivalently, the goal is to maximize the excess of the value of the goods to the consumer (consumer surplus plus revenue) over the resource cost of producing them. Before proceeding to find this maximum we must analyze the consumer surplus generated by our monopolistically competitive market.

3.1. Consumer Surplus

One measure of the consumer surplus is the triangular area under the firm’s dd’ demand curve; this measure, which is sensitive to \( \overline{p}_i \), is

\[
S_{dd}^i = d_0(d_1-p_i + d_2 \overline{p}_i)^2/2c(n). \tag{30}
\]

This would be the relevant measure for considering how consumers are affected when one firm adjusts its price, given the prices charged by all the other firms. For example, it would be the appropriate measure for evaluating the reduction in consumer surplus resulting from an increase in the price of strawberries, given the price of cherries and raspberries; but it is not relevant when all prices in the industry shift, as when moving to an alternative monopolistic competitive equilibrium; and it is not appropriate for measuring the effect of a change in the number of products. The required measure is derived by considering the area under the DD’ curve, which shows the benefit from consuming one product when all firms are charging the same price; i.e., \( p_i = p_j = \overline{p} \).

For the ith firm we have

\[
S_{DD}^i(\overline{p}, q_i) = [d_i/(1-d_2) - \overline{p}]q_i/2 = [d_i - (1-d_2) \overline{p}]^2n/c(n). \tag{31}
\]

For the entire market,

\[
S_{DD}^D(\overline{p}, n) = \sum S_{DD}^i(\overline{p}, q_i) = [d_i/(1-d_2) - \overline{p}]Q(\overline{p}, n)/2, \tag{32}
\]

where \( Q(\overline{p}, n) \) is given by (8).

3.2. Total Benefit

The total benefit generated by the industry when selling \( n \) differentiated items at average price \( \overline{p} \) is the sum of consumer surplus plus industry profit, which is

\[
B(\overline{p}, n) = S_{DD}^D + \pi = \{[(d_i/(1-d_2) + \overline{p})]/2 - k_i\}Q - nk_0. \tag{33}
\]
Properties 7 and 8 imply that Q increases but less than proportionately to n; since the last term in (33) decreases in proportion to n, for sufficiently large n, B(\(\bar{p}, n\)) must decline when more products are introduced to the market. But will B(\(\bar{p}, n\)) attain its maximum for a larger or smaller n than that determined by free entry and exit?

A wide range of possibilities for the effect of n on the relationships among consumer surplus, profit, and total benefits is suggested by the four panels of Figure 2.

- Panel 1 reports the situation for the Nash equilibrium price of $12. Industry profits (indicated by the bottom curve of the panel) are maximized with 25 firms in the industry, but free entry will lead to the monopolistic competitive solution with 100 firms realizing zero profit at point e. Unfortunately, this monopolistically competitive equilibrium has too little product diversity. The benefit function showing the sum of consumer surplus plus profits (heavy line on graph) is maximized with 225 different firms in the industry, point m, given that \(\bar{p} = $12\). But this would involve industry profits of -$4,800. In order to induce the benefit maximizing number of firms to enter the industry, the government would have to pay a subsidy of $21.33 to each firm.

- Panel 2 shows the marginal cost pricing solution, \(p = k_1\). With a price ceiling of $4.00 no firm will remain in the industry and output will be zero. A subsidy covering fixed costs of \(k_1 = $64\) for each of 244 firms, or a total of $15,616, would maximize total benefits at point m.\(^7\)

- Panel 3 presents the opposite case in which the government imposes a price floor of $24 so as to maximize the combined profits of all the firms in the industry. This certainly will improve the profit situation, for any given number of firms in the industry; but in the absence of effective barriers to entry, the number of firms will eventually grow to 244, driving economic profit to zero. Given the price floor of $24, profits would be maximized if the number of firms were restricted to 61. Benefits would be maximized, given the $20 price floor, with 137 firms in the industry.

- Panel 4 is similar to the first, with \(\bar{p} = $12\). but with a less elastic response of industry demand to changes in the number of firms. Now the Benefit Maximizing n is to the left of the zero profit n. There is too much product variety!

**Figure 2: Optimal n, given**

Under what conditions will a monopolistic competitive market yield too little rather than too much product variety in long-run equilibrium? Assuming constant elasticity of Q with respect to n (i.e., \(c(n) = n^{1/\gamma}\)), and invoking a continuous approximation, we have \(\partial Q/\partial n = \gamma Q/n\), and so

\[
\frac{\partial B}{\partial n} = \left[\frac{d_1}{2} (1-d_2) + \frac{\bar{p}}{2} - k_1\right]\gamma Q/n - k_0. \tag{34}
\]

With free entry, profits will be zero in the long run, implying that in monopolistic competitive equilibrium we must have \(Q^* = nk_0/(\bar{p} - k_1)\). Evaluated at the equilibrium (zero \(\pi\)) level of output,

\[
\frac{\partial B}{\partial n} = \left[\frac{d_1}{2} (1-d_2) + \frac{\bar{p}}{2} - k_1\right]\gamma k_0/(\bar{p} - k_1) - k_0. \tag{35}
\]

\(^7\) As will be explained in a moment, a $10 per unit subsidy would also achieve a solution characterized by marginal cost pricing and the optimal number of firms, provided there is free entry into the industry.
If $\partial B/\partial n$ is positive at equilibrium output, benefits are still increasing with $n$, implying that free entry has led to insufficient product variety. If this derivative is negative, free entry has led to excessive product diversity. In evaluating this issue it is useful to note that for $p > k_1$ there is a critical value of the elasticity of the response of industry sales to changes in product variety:

$$\gamma^0 = \frac{2(p - k_1)}{d_1/(1 - d_2) + p - 2k_1}$$

(36)

If $\gamma = \gamma^0$, $\partial B/\partial n = 0$ and monopolistic competition has generated the optimal degree of product variety. If $\gamma$ is below this critical value, there is too much product variety. If $\gamma$ exceeds this value, the monopolistic competitive equilibrium has insufficient product diversity. Note that $d_1/(1 - d_2) > p > k_1$; i.e., price is less than the intercept of inverse market demand function (9) but greater than marginal cost. Therefore, $0 < \gamma^0 < 1$. Also, $\partial \gamma^0/\partial k_1 > 0$, $\partial \gamma^0/\partial d_1 < 0$, and $\partial \gamma^0/\partial d_2 < 0$. 
In the absence of empirical parameter estimates, there is no basis for a presumption that markets usually generate either too much or too little product diversity. We can say that other things being equal, if the elasticity of the response of industry sales ($\gamma$) to increased product variety is sufficiently small, there will be excessive product diversity. Since it is not unreasonable to suppose that gasoline sales are insensitive to the number of retail outlets while eating out is likely to be quite responsive to the variety of restaurants in the market area, one might suspect that other things being equal, there may be more than the optimal number of gasoline stations but fewer than the optimal number of ethnic restaurants.
4. Taxing implications

It is best to start the discussion of applications of the theory by considering the incidence of lump-sum taxes and excise taxes. Various results will be summarized on Table 3. We find that the effects of a tax under monopolistic competition can be quite different from what happens under competition or under monopoly. We shall also find that subsidies may be employed to establish the optimal number of firms in the industry.

Table 3: Summary Results

4.1. Lump-sum tax or a licensing fee

A lump-sum tax imposed on each firm is equivalent to an increase in $k_0$. This parameter only influences the equilibrium number of firms in the industry – it has no effect on price but it will influence the degree of product variety. For our numerical example, the imposition of a $64 lump sum tax would lead to a reduction in the number of firms surviving in monopolistic competitive equilibrium from 100 to only 25 (See line 3 of Table 3). More generally, equation (24) reveals that for $c(n) = n^{1-\gamma}$, the effect of a lump sum tax $t_0$ on an industry initially in equilibrium with $n$ firms is to change the number of firms to

$$n_t = n\left(k_0/(t_0+k_0)\right)^2.$$ (37)

Like monopoly but unlike the competitive model, the lump-sum tax does not affect price in the long run. But unlike monopoly, there is no decline in economic profit, which is zero both before and after the tax for each individual firm as well as for the industry. The entire burden of the lump-sum tax arises from the loss imposed upon consumers by the reduction in product variety.

4.2. Excise Tax

The effect of an excise tax is slightly more complicated. If we let $p_i^* = p_i - t_0$ denote the price received by the seller, net of the tax, then from inverse demand function (4) we find that

$$p_i^*(q_i, p, n, t_0) = d_1 - t_0 + d_2 p - q_c(n)/d_0,$$ (38)

which means that the analysis is expedited by substituting $d_1^* = d_1 - t_0$ throughout. From equation (16) we find that the immediate response of profit maximizing firms, given $p_i^*$ and $n$, will be to pass 50% of the excise tax directly on to the consumers, as recorded on line 4 of Table 3. This is identical to the result for a profit-maximizing monopolist with a linear demand curve and constant marginal costs, but this is not the end of the story in the case of monopolistic competition. The short-run effect, given the number of firms but recognizing the change in $p$, is to push up the Nash equilibrium price the consumer pays by $t_1/(2-d_2)$, as may be seen from equation (17); i.e.,

$$p_{N_i}^* = (d_1 + t_1 + k_1)/(2-d_2) = p_{N}^* + t/(2-d_2).$$ (17)*

For our numerical example, a $4.00 excise tax yields a market price (gross of the tax) of $15.20 (Line 5). Output drops to 7.2 and losses rise to $12.16 at each firm. Since that is the best they can do, firms facing negative profits will exit the industry.

Now the number of firms must adjust in the long run to restore zero profits, but this does not involve any further change in market price. First we solve for the output of each firm that will yield zero profits at equilibrium price $p_{N_i}^*$. Since profits will be $\pi = (p_{N_i}^* - t_1 - k_1) q_{N_i}^* - k_0 = 0$,

\[ 8 \text{ This result from the assumption that marginal cost is constant and the demand curve is linear (or alternatively, has constant elasticity).} \]
## Table 3: Summary Results

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1. With 25 firms, the Nash equilibrium yields positive profit, encouraging entry into the industry.
2. With 100 firms, profits are driven to zero, establishing a happy monopolistically competitive equilibrium.
3. Lump sum tax of $64 per firm yields the Monopolistic Competitive equilibrium with 25 firms; the burden of the tax is in the form of reduced product variety.
4. Half of an excise tax of $4.00 would be passed on to a firm’s customers, given
5. When other firms raise their prices in response to the $4 excise tax, the eventual Nash equilibrium more of the B71 excise tax being passed on to consumers
6. With the $4 excise tax, monopolistic equilibrium will only be restored when the number of firms shrinks to 65. (65.61)
7. With marginal cost pricing, welfare is maximized with 244 firms.
8. With a price of $12, welfare is maximized with 229 firms.
9. Joint Profits are maximized with 61 firms in the industry.
10. A subsidy of $10 per unit of output will lead to a monopolistic competitive equilibrium with marginal cost pricing of $4.00 and 244 firms in the industry.
For our numerical example, we have \( q^N_i = 64/(15.20-4-4) = 8.888 \) (Line 6). To find the equilibrium number of commodities, substitute (17)* into equation (6) to obtain

\[
c(n)^N = d_0[d_1 - (1-d_2) \bar{p}]/q^N_i,
\]

or if \( c(n) = n^{1-\eta} \), then

\[
n = \left\{ d_0[d_1 - (1-d_2) \bar{p}]/q^N_i \right\}^{1/(1-\eta)}.
\]

Equation (17)* also reveals the size of the subsidy per unit of output (i.e., a negative excise tax) that would have to be awarded to producers in order to induce them to price at marginal cost, as required for efficiency; i.e., \( p^N = k_1 \). For our numerical example with \( d_2 = 0.75 \) and \( p^N = $12 \), we find that a subsidy of \$10 would be required. In the short run the price charged to consumers would exceed \$4 and producers would reap positive profits, but new firms would enter as long as positive profits were being made. The long run zero profit equilibrium would involve a price to consumers of \$4.00 with 244 firms in the industry and output per firm of 6.4 (Line 7).

5. Increase in the minimum wage and other cost pressures

According to the conventional wisdom, hikes in the minimum wage take jobs away from low-income workers. David Card and Alan B. Krueger [1995] challenged this presumption in their classic “event study.” They compared the effects on employment in fast-food restaurants of changes in the difference between the minimum wage in New Jersey and that in Pennsylvania. Their primary focus was on the effects of the minimum wage on employment at individual retail outlets. The minimum wage is similar to an excise tax in that it increases the cost of producing each unit of output, the parameter \( k_1 \). An increase in long-run employment at each retail outlet, which might be called the “Card-Krueger” effect, is predicted by the theory of monopolistic competition.

Responses to the Card-Krueger study focused largely on the question of the appropriateness of the sample evidence, but there is a more fundamental problem that may limit the implications of their study. The theory of monopolistic competition suggests that the number of retail outlets will decline, and the decline will be large enough to more than offset the increased sales and employment at each outlet. Employment increases at a diminishing number of fast-food establishments as a result of the rise in the minimum wage, but the number of firms effect dominates the size-of-firm effect. Card and Krueger’s evidence about the effect of the minimum wage on the number of outlets was much more limited than their event-study evidence from panel data on the size of firm effect. The only evidence was provided by fifty observations of statewide data on restaurant openings for the McDonald chain. They report four regressions involving alternative specifications that yield positive coefficients for the minimum wage as a determinant of the proportionate change in the number of restaurants in the 50 states, but only one of these was significant.\(^9\) If the number of firms in fact declines, the model of monopolistic competition reconciles the Card-Krueger event-study finding that employment tends to increases at each firm with the conventional view that unemployment will increase.

Other forms of cost-push pressure may generate effects similar to a hike in the minimum wage. For example, an OPEC generated increase in the price of petroleum may lead to an in-

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\(^9\) Card and Krueger [1995, pp 53-65] also report four other regressions based on the number of MacDonalds openings in each state, but because of substantial variation among states in population, these regressions are obviously distorted by heteroscedasticity. In [1998] they do discuss in some detail the number of outlets, but only as it affects the validity of their sampling procedure.
crease in sales at each gas station but a reduction in the number of outlets and a fall off of total industry sales.

6. General Equilibrium

It is necessary to recast the analysis of monopolistic competition in a general equilibrium framework characterized by equilibrium in the labor and product markets and complicated by government spending and taxes.

In order to take into account the optimized work-leisure choice and the effects of dividend distributions and taxes, let us write the budget constraint facing the $j$th consumer as

$$w(24-Y_j) - T_j + \pi_j = \sum P_i q_{ij}, \tag{42}$$

where $w$ is the wage rate, $Y_j$ is hours of leisure (now the numéraire) enjoyed by the $j$th worker, $T_j$ is a lump-sum tax, $\pi_j$ is our citizen’s share in corporate profits, the $P_i$ are (nominal) prices, $q_{ij}$ is the quantity of the $i$th good purchased by the $j$th consumer, and there are 24 hours in the day.\(^{10}\)

Letting $H_j = 24-Y_j$ denote the hours worked by the $j$th citizen and $m$ the number of citizens, the supply of labor (the only resource used in the production process) is

$$H = \sum_{j=1}^{m} H_j = \sum_{j=1}^{m} (24-Y_j). \tag{43}$$

Market clearing in the labor market requires that labor supply equal the total labor required in the production of all the goods purchased by consumers ($q_i = \sum_{j} q_{ij}$) and the government ($g_i$):

$$H = nk_0 + k_1 \sum_{i=1}^{n} (q_i + g_i). \tag{44}$$

It is also necessary in equilibrium for the sum of any dividends (profit shares) received by the citizens ($\pi_j$) to add up to total business profits:

$$\sum_{j=1}^{m} \pi_j = \sum_{i=1}^{n} (P_i - k_1)(q_i + g_i) - nk_0. \tag{45}$$

Because only relative rather than absolute wages and prices are determinate in this model, an hour of labor is the appropriate unit of account.

6.1. The balanced budget multiplier

Now suppose that an initial monopolistic competitive equilibrium is disturbed by the imposition of a lump-sum tax $T > 0$ in order to finance government purchases of $g = \sum g_i$ units of output from established supplier at the prevailing market price, perhaps for the military or to aid in recover from flood or earthquake or some other disaster. We will find that a new equilibrium exists with output up by the amount of government spending but with no change in the real wage. To see why, let $\overline{p}^o$, $Q^o$, $H^o$ and GDP$^o$ denote the price level, output, hours worked, and Gross Domestic Product in the initial equilibrium. Let $g_i$ denote the number of units of output purchased from the $i$th firm. Now suppose as a result of government spending $G = \sum g_i$, financed with taxes $T$ we have $\overline{p} = \overline{p}^o$, $Q = Q^o + G$, $H = H^o + \sum g_i k_1$ and GDP = GDP$^o + G$. Then the new situation can be shown to be a monopolistic competitive equilibrium. Note first that since the

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\(^{10}\) Equation (42) can be recast into the same form as (11) in the appendix by letting $M = 24w - T + \pi = \sum P_i q_{ij} + wY$. 
marginal utility of leisure is constant and the real wage \((1/\bar{p})\) remains the same, consumers increase their work effort by however much is required in order to consume precisely the same consumption bundle as before. Firms now make positive profits because the prevailing price at which they sell the extra output is above marginal cost. Profits will equal the excess of the amount spent on procurement by the government \((G=T)\) over the cost of the labor hours required to produce it. Since the profits are distributed as dividends to worker-stockholders, the income of workers, after tax, is precisely the same as before. Because the government purchases only from established firms, it will not pay for new firms to enter the market. Thus the increase in government spending on goods and services that is financed entirely by additional taxes has led to an equal increase in GDP, as predicted by the Balanced Budget Multiplier proposition of Keynesian theory.\(^{11}\)

To illustrate, let us elaborate on our earlier numerical example, using the same demand and cost parameters as before and with \(n = 100\) and \(m = 1,600\).\(^{12}\) GDP and profits will be measured in hourly wage units, which will be indicted by a dollar sign.

**Initial Equilibrium:**

\[
\bar{p} = p_i = \$12; q_i = 8, Q = 800, \text{ and } R = p_iQ = \$9,600; \ G = T = 0
\]
\[
C_i = \$64 + \$4 \times 8 = \$96; \ H = 6,400 + 4 \times 800 = 9,600
\]
\[
h_i = 5.5; \ m = 1,600, \ n = 100; \ \pi = R - C = 0; \ GDP = H + \pi = \$9,600.
\]

Now suppose that this equilibrium is disturbed by the government purchase of 300 units of output at prevailing market price \(p_i = \$12\). Suppose the spending is financed by a lump sum tax yielding \$3,600, or \$2.25 per person. In the new equilibrium the representative firm will produce \(q_i = 11\), incur costs of \$108, realize revenue of \$132, and enjoy profits of \$24, which are distributed to citizen shareholders. Thanks to the constant marginal utility of leisure, the real wage is unchanged at \(1/\bar{p} = 1/12\). Now each citizen is paid for working a 6.8 hour day and also receives a \$1.50 profit share. After taxes, per capita income is at precisely the same level as before. Because the marginal utility of leisure is constant, workers have responded by working an additional hour per day but they are consuming precisely the same bundle of goods as before.

**Balanced Budget Multiplier equilibrium: government pays prevailing market prices**

\[
\bar{p} = p_i = \$12; q_i = 11, Q = 1,100, \text{ and } R = p_iQ = \$13,200; \ G = T = \$3,600
\]
\[
C_i = \$64 + \$4 \times 11 = \$108; \ H = 6,400 + 4 \times 1,100 = 10,800
\]
\[
h_i = 6.8; \ m = 1,600; \ n = 100; \ \pi = R - C = \$2,400; \ GDP = H + \pi = \$13,200.
\]

Observe that the national income accounts will report that the value of output is up by \$3,600, precisely the amount of government spending. Thus, the balanced budget multiplier is unity. However, disposable income is precisely the same as before, and is equal to the unchanged value of consumption spending. While the national income accounts will report that workers enjoy the same disposable income as before, they are worse off because they are putting in longer hours.

The balanced budget multiplier story is slightly different if the government puts its purchase of 100 units out to competitive bidding rather than paying the prevailing market price. Competition among firms will push the bids down to marginal cost of \$4.00 per unit. Because

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\(^{11}\) Cf Henry Wallich[1944] and the literature there cited.

\(^{12}\) The values of the utility function parameters required to generate demand of \(q_i = 8\) when \(p_i = \$12\) are specified at the end of the Appendix.
the government can procure at this bargain price, a lower tax will be required but economic prof-
its will be zero. While the increase in measured output is less, so is the wage unit magnitude of G;
thus the balanced budget multiplier is again unity. Apart from possible distributional differences,
the net effect of the government procurement on the citizenry is the same.

Balanced Budget Multiplier equilibrium: competitive bidding on the government contract:

\[
\bar{p} = p_i = $12; \ q_i = 11, \ Q = 1,100, \text{ and } R = k_iq + \bar{p}(Q-g) = $10,800; \ G = T = 1,200
\]

\[
C_i = $64 + $4\times11 = $108; \ H = 6,400 + 4\times1,100 = 10,800,
\]

\[
H_i = 6.8; \ m = 1,600, \ n = 100, \pi = R - C = 0; \text{ GDP } = H + \pi = $10,800.
\]

This demonstration that the balanced budget multiplier is unity required several restric-
tive assumptions: The revenue to finance the increased expenditure had to be raised with a lump
sum tax. Also, the result wold be invalidated if either the marginal utility of leisure or marginal
cost were not constant. Further, it was assumed that the demand curve is linear.

6.2. Subsidizing marginal cost pricing:

Suppose, once again, that the government establishes a lump sum tax on the citizenry, but
that it uses the revenue as a subsidy to induce enterprises to price at marginal cost. We must
elaborate on the partial equilibrium analysis of Section 4.2 by recognizing that the subsidy in-
duced reduction in price (measured in labor units) means that workers will enjoy a higher real
hourly wage. It is necessary to take account of the fact that higher real wages may induce a
change in the number of hours that workers will want to spend on the job. The substitution effect
will lead to an increase in labor force participation. Thanks to the constant marginal utility of
leisure assumption, there is no income effect.

The consequences of using a lump sum tax to finance a marginal-cost pricing subsidy are
illustrated with our hypothetical numerical example:

Subsidized Marginal Cost Pricing

\[
\bar{p} = p_i = $4; \ q_i = 6.4, \ Q = 1,561.6, \text{ and } R = p_iQ = $13,200; \ G = 0, \ T = $6,400
\]

\[
C_i = $64 + $4\times6.4 = $89.6; \ C = 89.6\times244 = $21,862
\]

\[
h_i = 13.7; \ m = 1,600, \ n = 244; \pi = R - C = 0; \text{ GDP } = H + \pi = $21,862.
\]

The subsidy has reduced the price of output from $12 to $4, implying a three-fold increase in the
real wage. As a result, there is a substantial expansion in the supply of labor and a marked in-
crease in measured GDP but less leisure. Welfare is enhanced because of the increase in con-
sumer surplus, now that output is priced at marginal cost, plus increased product variety. The
partial analysis of the subsidy in Section 6.2 was incomplete because it left out the effect of the
price reduction on the real wage, the resulting increase in hours worked, and the expansion of
output.

7. Conclusion

This paper has modeled monopolistic competition with a linear demand function derivable
from a utility function of a maximizing consumer. As with many other models of monopolistic
competition, it is assumed that the other good has constant marginal utility of income and that
marginal cost is constant. We found:
• A monopolistic competitive market is likely to generate too much product diversity if an increase in product variety has little impact on total industry sales, other things being equal. The opposite form of market failure, too little product variety, is likely to result when demand responds strongly to increased variety. Equation (36) specifies the determinants of the critical value of the elasticity of market response to changes in the number of products for which the monopolistic competitive equilibrium yields the appropriate number of firms.

• The incidence of various types of taxes on a monopolistically competitive market was found to differ from the predictions of both the competitive and the monopoly model. A lump sum tax or licensing fee does not lead to a change in price or economic profit, but it does place a burden on consumers by reducing the degree of product variety. An excise tax leads in the long run to higher prices and an increase in the output of each firm, but both product variety and total industry sales decline.

• An increase in the minimum wage is also likely to lead to an increase in employment at individual firms, but the firm size effect is swamped by a reduction in the total number of firms and as a result, industry employment is reduced. Similarly, an OPEC induced increase in petroleum prices may lead to a smaller number of larger gas stations.

• A general equilibrium model of monopolistic competition shows that the financing of an increase in government spending on goods and services with a lump-sum tax leads to an increase in GDP of equal size, as predicted by the “balanced budget multiplier” proposition of Keynesian theory. This is so whether the government purchases at prevailing market prices from established suppliers or achieves a discount below the prevailing market price by putting its purchase out for competitive bidding.

• When a subsidy financed by a lump-sum tax is used to induce firms to price at marginal cost, the real wage will increase and both measured GDP and hours worked may be up substantially in the new general equilibrium. Welfare will be enhanced by greater product variety as well as the establishment of marginal cost pricing.

Appendix

Proof of Theorem 1:

By Property 9, the inverse demand function may be written for any market size n as

\[ p = d_1 + d_2 \bar{p}_i - dq \]  \hspace{1cm} (1)

and for size n+1 as

\[ p' = d'_1 + d'_2 \bar{p}_i - d'q'. \]  \hspace{1cm} (2)

An indirect proof will now establish that for all \( \bar{p}_i \)

\[ d_1 + d_2 \bar{p}_i = d'_1 + d'_2 \bar{p}_i, \]  \hspace{1cm} (3)

which implies that for \( p_i = d_1 + d_2 \bar{p}_i, \) \( q_i(p_i, \bar{p}_i, n) = q_i(p_i, \bar{p}_i, n+1) = 0 \)

Suppose in contradiction to (3) that for some \( \bar{p}_i \) and n

\[ d_1 + d_2 \bar{p}_i > d'_1 + d'_2 \bar{p}_i, \]  \hspace{1cm} (4)

implying, with Property 4, that for \( p'_i = d'_1 + d'_2 \bar{p}_i, \)
\[ q_i(p_i', \bar{p}_i, n) > 0 \text{ while } q_i(p_i', \bar{p}_i, n+1) = 0; \]  
(5)

but this contradicts Property 7.

Alternatively, suppose that

\[ d_1 + d_2 \bar{p}_i < d_1' + d_2' \bar{p}_i, \]  
(6)

implying that for \( p_i = d_1 + d_2 \bar{p}_i \)

\[ q_i(p_i, \bar{p}_i, n) = 0 \text{ while } q_i(p_i, \bar{p}_i, n+1) > 0; \]  
(7)

but this contradicts Property 6.

Therefore, (3) holds for all \( \bar{p}_i \), implying \( d_1 = d_1' \) and \( d_2 = d_2' \).

Thus our inverse demand function must be of the form

\[ p = d_1 + d_2 i_p - d(n)q, \quad n = 1, 2, \ldots \]  
(8)

where only the coefficient of \( q \) can depend on the number of firms.

Let the function \( d(n) \), defined on the positive integers, generate a sequence of coefficients satisfying Properties 6 and 7. Then for any conveniently specified scalar \( d_0 \), let \( c(n) = d_0 d(n) \) and we obtain equation (4). Equation (3) follows immediately.

Total market sales are

\[ Q(\bar{p}, n) = \sum_{i=1}^{n} q_i(p_i, \bar{p}_i, n) = d_0 (nd_1 - \sum p_i + d_2 \sum \bar{p}_i) / c(n) = nd_0 [d_1 - (1-d_2) \bar{p}] / c(n), \]  
(9)

Theorem 2:

Consider a consumer with utility function,

\[ U(q_1^c, \ldots, q_n^c, Y) = u_0 + u_1 \sum q_i^c - u_2 \sum q_i^c^2 / 2 - u_3 \sum \sum q_i^c q_j^c / 2 + Y, \quad (u_1 > 0, u_2 > 0, u_3 > 0), \]  
(10)

where the \( q_i^c \) are the quantities consumed of \( n \) goods, \( Y \) is a numéraire type good yielding constant marginal utility, and it is understood that the parameters \( u_0, u_1, u_2, \) and \( u_3 \) may depend on \( n \). If our consumer is a price-taker maximizing this utility function subject to budget constraint

\[ M = \sum P_i q_i^c + P_Y Y, \]  
(11)

where \( M \) is money income, the demand equation will be:

\[ q_i^c = u_i / (u_2 + nu_3) + [nu_3 / (u_2 + nu_3) u_2] \bar{p} - (1/u_2)p_i, \quad \text{for } i = 1, \ldots, n. \]  
(12)

Proof: \( L(q_1, \ldots, q_n, Y, M, \lambda) = u_0 + u_1 \sum q_i^c - u_2 \sum q_i^c^2 / 2 - u_3 \sum \sum q_i^c q_j^c / 2 + Y + \lambda(M - \sum P_i q_i^c - P_Y Y). \)  
(13)

Now for a maximum we must have

\[ \partial L / \partial Y = 1 - \lambda P_y = 0 \text{ or } \lambda = 1 / P_y \]

and

\[ \partial L / \partial q_i^c = u_i - u_2 q_i^c - u_3 \sum q_i^c - p_i = 0, \quad (i = 1, \ldots, n), \]  
(14)

where \( p_i = P_i / P_y = \lambda p_i \) is the price of good \( i \) relative to that of good \( Y \). From (14) we obtain

\[ q_i^c = u_i / u_2 - u_3 \sum q_i^c / u_2 - p_i / u_2 \]  
(15)

As a first step toward finding the demand equation, we sum both sides to obtain:
\[ \sum q^e_j = nu_1/u_2 - nu_3 \sum p_i/u_2 = n(u_1 - \bar{p})/(u_2 + nu_3). \]  
\[ (16) \]

Substituting back into (15) yields
\[ q^e_i = u_1/u_2 - nu_3 (u_1 - \bar{p})/(u_2 + nu_3)u_2 - p_i/u_2, \]
\[ (17) \]
which simplifies to the linear demand equation system
\[ q^e_i = u_1/(u_2 + nu_3) + [nu_3/(u_2 + nu_3)u_2] \bar{p} - (1/u_2)p_i, \quad i = 1, \ldots, n. \]
\[ (18) \]

Substituting (12) into (11) leads to the demand equation for \( Y \).

Note that (12) neglects sign constraints; it is valid unless \( M \) is so low that (11) yields a negative value for numéraire good \( Y \) or \( p_i \) is so high that the consumer is priced out of the market.

If there are \( m \) consumers with identical tastes, the demand function for the total sales of each product, obtained by multiplying (12) by \( m \), will be identical to demand function (3) of the text with coefficients \( d_0/c(n) = m/u_2 \), \( d_1/c(n) = nu_1/(u_2 + nu_3)u_2 \), and \( d_2/c(n) = mnv_3/(u_2 + nu_3)u_2 \). Or given the demand coefficients, the utility function parameters are \( u_1 = d_1/(1-d_2) \), \( u_2 = c(n)/md_0 \), and \( u_3 = u_1u_2d_2/md_1 = c(n)d_3/[d_0(1-d_2)mn] \). For example, if \( c(n) = \sqrt{n} \), \( d_0 = 10 \), \( d_1 = 11 \), \( d_2 = 0.75 \), \( n = 100 \) and \( m = 1,600 \), then we have \( u_1 = 44 \), \( u_2 = 0.000625 \), and \( u_3 = 0.00001875 \).
References


